Multi-Channel Sequential Search with Application to Piracy

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Abstract

This paper presents a multi-channel search model, where each channel consists of multiple firms that are ex ante homogeneous to consumers. Consumers, nevertheless, ex ante can tell channels apart in the following aspects: search costs, product quality, product availability and price distributions. We first analyze the channel selection problem, i.e. which channel a consumer searches first. We show that a consumer’s optimal channel selection at any stage of the search is independent of found prices – in other words, history independent. We further show that the optimal channel choice can be determined using each channel’s reservation price and consumer valuation. One implication is that interleaved search never takes place.

We then apply this model to study the competition between legitimate and piracy products. One immediate result is that reducing the number of piracy services has no impact on consumers’ decision of whether to pirate unless all piracy services are shutdown. When piracy products pose a threat, legitimate firms may respond by giving up low search cost consumers. This leads to the surprising result that piracy threat may induce firms to increase, rather than to decrease, their product prices. At the same time price distribution among legitimate firms can be more polarized in that the variance in prices increases.

Keywords: search, channel competition, information good, piracy, price dispersion
Imagine a consumer who wants to own the Blu-ray version of a movie. One option this consumer has is to search retailers in the legitimate online channel, such as BarnesNoble.com and Amazon.com, until she finds an acceptable price. Alternatively, she can visit services in the piracy channel, such as mininova.org and thepiratebay.org, where she may find a piracy version with acceptable quality for free. This second scenario is what many creators and distributors of information goods have been fighting against. However, despite years of costly efforts by trade groups such as the RIAA, piracy remains largely untamed: a joint research by the Business Software Alliance and IDC (2006) puts the piracy rate at 35% worldwide for 2004-2006.

A number of papers in economics and information systems have touched the piracy issue theoretically (Sundararajan 2004, Chellappa and Shivendu 2005, Park and Scotchmer 2005, Peitza and Waelbroeck 2006, Belleflamme and Picard 2007). Nevertheless, to date none has considered the (often true) possibility that both the legitimate online channel and the piracy channel include many retailers/services, and thus competition can exist both inside and across channels at the same time. Our research fills in this theoretical gap by proposing a multi-channel sequential search model. In our model, consumers face multiple channels (such as the legitimate and piracy channels), where each channel consists of multiple firms. To obtain information such as price and product availability, consumers incur a cost for each firm searched. All firms in the same channel are ex ante homogenous to consumers, thus consumer search within a channel is random. Consumers, nevertheless, ex ante can tell channels apart in the following aspects: search costs, product quality, product availability and price distributions.
We carry out our analysis in two parts. In the first part, we consider a general multi-channel search model that is motivated yet broader than the aforementioned piracy story. Therefore, the results are also applicable to other business scenarios of channel competition, such as competition between online and offline channels. Our key research question in this part is the channel selection problem: given a search history, which channel a consumer will search next. A surprising and strong result we find is that a consumer’s optimal channel selection at any stage of the search is independent of found prices – in other words, history independent. We further show that the optimal channel choice is uniquely determined by every channel’s reservation price and consumer valuation: a consumer always first searches the channel that offers the largest difference between consumer valuation and the channel’s standalone reservation price.

Because a consumer’s channel selection is history independent, interleaved searches across channels never takes place: a consumer always exhausts firms in her preferred channel before moving on to the second-preferred channel.

In the second part of the analysis we focus on the legitimate and piracy channels case. The key research question is how the existence of piracy services affects pricing and demand of legitimate retailers. A striking result we find is that reducing the number of piracy services has no impact on consumers’ decision of whether to pirate unless all piracy services are shutdown. Intuitively, as long as at least one piracy service remains, our aforementioned analysis predicts that consumers will not alter their channel preferences. Therefore, those who prefer piracy still do so no matter how many piracy services are shutdown (unless of course all are shutdown).

Compared to the case of no piracy, the existence of piracy products may increase the expected prices that legitimate retailers charge. This seemingly counter-intuitive result can be explained as follows. When piracy threat is strong enough, legitimate retailers will give up
consumers with low search costs. As a result, high search cost consumers are left within the legitimate channel, which actually reduces in-channel competition among retailers. As a result, retailers are able to charge higher prices. Also interestingly, we find the existence of piracy products can lead to polarized pricing among legitimate retailers – more retailers end up charging a very high price.

Our work is related to the piracy literature (Peitza and Waelbroeck 2006). The economics and information systems research communities have approached piracy control (including whether it is necessary) from a number of perspectives. One school of research assumes that pirated goods have lower qualities than their legitimate counterparts, thus firms can use quality differentiation for piracy control (see, for example, Sundararajan 2004, Chellappa and Shivendu 2005, Park and Scotchmer 2005, Belleflamme and Picard 2007). A related second school of research argues that piracy control may not be necessary since the network effects created by piracy goods can increase the demand pool and thus benefit sales of legitimate goods (see, for example, Bomsel and Geffroy 2005, Chellappa and Shivendu 2005). Our work differs in that we consider both multiple legitimate retailers and multiple piracy services, and thus we can address previously unanswered questions such as how the existence of piracy affects price distribution among legitimate retailers.

Our work also adds to the sequential search literature (Diamond 1971, Reinganum 1979, Burdett and Judd 1983, Stahl 1989, Baye et al. 2006), where the last reference provides a comprehensive survey of search models and empirical findings. A common feature of previous work on sequential search is that all firms are ex ante homogeneous to consumers; in other words, they consider only a single channel. Our work differs in that we consider multiple channels, consumers’ channel choices and inter-channel competition.
In modeling price dispersion, we follow the approach pioneered by Stahl (1989), in which consumers are assumed to have two types: shoppers and non-shoppers. The former is assumed to have no search costs. We pick this approach not only because Stahl’s model is widely accepted in research on sequential search, but also because new information technologies, such as cell phone-based price comparison software, have dramatically reduced search costs for many consumers – examples of shoppers. Stahl (1989) considers a single channel while we discuss cross-channel competition.

The rest of the paper is organized as follows. Section 2 lays out the general multi-channel search model. Section 3 analyzes optimal consumer search behavior. Section 4 discusses piracy control and the impact of piracy on legitimate firm pricing. Section 5 concludes this paper.

2. The Model

There is a continuum of consumers with mass normalized to one, each of whom wants at most one unit of a product. A consumer’s valuation of a product is $qv$, where $q$ represents the quality of the product offered by a firm, and $v$ represents the heterogeneity among consumer valuations. Following the sequential search literature, we assume that all consumers’ valuations are high enough so that they will search at least one firm (Diamond 1971, Stahl 1989, Baye et al. 2006).

There are a number of firms that may differ in the following aspects: some carry the product while others do not; some offer a better quality than others; and some charge a higher price than others. Except for quality which is common knowledge, we assume that ex ante consumers know neither product availability at nor price offered by any firm. As a result, a consumer will have to search a firm if she wants to learn about availability and price at this firm.
Due to the above information asymmetry, *ex ante* consumers can only identify firms into two sets, which we refer to as *channels a and b*.\(^1\) All firms within channel \(i \ (i = a, b)\) are *ex ante* homogenous from a consumer’s perspective: they have the same quality \(q_i\); they have the same likelihood \(\eta_i\) of carrying the product; and their prices follow the same *ex ante* price distribution \(F_i(p_i)\) on support \([\underline{p}_i, \overline{p}_i]\). \(\overline{p}_i > \underline{p}_i \geq 0\). If the density of \(F_i(p_i)\) exists, denote it as \(f_i(p_i)\). Let \(A\) \((B)\) represent the number of firms in channel \(a\) \((b)\). Let \(c_i\) denote the per-firm search cost in channel \(i, \ i = a, b\). For ease of exposition, we normalize the marginal product cost to zero.\(^2\)

The timing of the model is as follows: the availability and quality of the product a firm carries are exogenously given; firms first make pricing decisions; consumers then decide how to search. In Section 3 we analyze consumers’ channel selection problem under any given price distributions. In Section 4 we narrow on to the legitimate/piracy channels case and analyze firm demand and pricing.

### 3. Consumers’ Channel Selection Problem

In this section we study consumers’ search behavior. We first solve a special case where one firm remains unsearched in each channel. We then generalize the results to any number of firms.

#### 3.1. One Firm Remains Unsearched in Each Channel

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\(^1\) This model and the analysis in the next section can be easily extended to any finite number of channels.

\(^2\) The model can be extended to include constant marginal costs; this will not qualitatively affect the results.
In this subsection we characterize a consumer’s optimal search behavior given that she has searched all but one firm in each channel. For \( i \in \{a,b\} \), let the lowest found price be \( \hat{p}_i \).

Without loss of generality, we assume \( q_a v - \hat{p}_a \geq q_b v - \hat{p}_b \), so that channel \( a \) offers the best deal so far. One implication is that, if the consumer stops searching now, she will purchase from channel \( a \) if \( q_a v - \hat{p}_a > 0 \), and not purchasing at all otherwise.

If the consumer continues searching, there are two possible search paths. The consumer can search the remaining firm in channel \( a \) (\( a \)-firm for abbreviation) first, and afterwards \( may \ or \ may \ not \) search the remaining firm in channel \( b \) (\( b \)-firm for abbreviation) – this is denoted as the solid path in Figure 1. We call this path the \( a \)-priority path. The dashed path in Figure 1 represents the case where the consumer searches channel \( b \) first, and is called the \( b \)-priority path. We call a consumer’s choice between \( a \)- and \( b \)-priority paths the \textit{channel selection problem}.

\[ q_a v - \hat{p}_a \] \text{channel} \( a \) \hspace{1cm} \text{Point} \( \alpha \)

\[ \text{channel} \ b \]

\[ \text{Point} \ \beta \]

\[ \text{channel} \ a \]

\[ \text{channel} \ b \]

\[ q_b v - \hat{p}_b \] \text{Point} \( \beta \)

\[ \text{channel} \ a \]

\[ \text{channel} \ b \]

\[ \text{Point} \ \alpha \]

\[ q_a v - \hat{p}_a \] \text{channel} \( a \) \hspace{1cm} \text{Point} \( \alpha \)

\[ \text{channel} \ b \]

\[ \text{point} \ \beta \]

\[ \text{channel} \ a \]

\[ \text{channel} \ b \]

\[ \text{Point} \ \beta \]

\[ q_b v - \hat{p}_b \] \text{channel} \( b \)

Figure 1. Two Search Paths When One Firm Left in Each Channel

We shall note that, because channels \( a \) and \( b \) may differ in product availability and price distributions, a consumer faces different information sets at points \( \alpha \) and \( \beta \) in Figure 1. Therefore, whether the consumer should continue searching after points \( \alpha \) or \( \beta \) and what the expected payoff will be are state-dependent. As a result, to study the channel selection problem,
for each path we need to consider the consumer’s total expected payoff from both steps of search. This differs significantly from previous research on single-channel search, where it is sufficient to study only the next immediate step of search to decide consumer behavior (Diamond 1971, Stahl 1989, Baye et al. 2006).

As a preparation, we first define the concept of cross-channel reservation price. For \(i,j \in \{a,b\}\), cross-channel reservation price \(r_j^i\) is the lowest price found in channel \(i\) such that searching in channel \(j\) will yield zero expected payoff. In other words, \(r_j^i\) satisfies

\[
\eta_j \int \limits_{\mathcal{P}_i} ((q_j - p_j) - (q_j - r_j^i)) f_j(p_j) dp_j = c_j.
\]

(1)

For notational convenience, denote \(H_i(\hat{p}_i) = \int \limits_{\mathcal{P}_i} (\hat{p}_i - x) f_i(x) dx\) for \(i \in \{a,b\}\), which represents the expected gain (before considering the search cost) of one more search in channel \(i\) conditioning on that the lowest found price in this channel is \(\hat{p}_i\) and that the firm to be searched carries the product. It is straightforward to verify that \(H_i'(\hat{p}_i) = F_i(\hat{p}_i)\). Therefore, \(H_i(\hat{p}_i)\) is a monotonically increasing and convex function. We can then rewrite equation (1) as

\[
\eta_j H_j((q_j - q_i) + r_j^i) = c_j.
\]

(2)

A special case is when \(i = j\), under which \(r_j^i\) solves

\[
\eta_j H_j(r_j^i) = c_j.
\]

(3)

\(r_j^i\) is the classical reservation price frequently cited in papers on sequential search within a single channel (Baye et al. 2006). Given that \(H_j(\bullet)\) is monotonic, a comparison of (2) and (3) yields the following result:
**Proposition 1:** For \( i, j \in \{a, b\} \), \( i \neq j \), cross-channel reservation price \( r_i^j \) and channel \( j \)
reservation price \( r_j^i \) satisfy the following condition:

\[
q_i v - r_i^j = q_j v - r_j^i.
\]

Proposition 1 is quite intuitive. In channel \( j \) alone a consumer should stop searching if she
finds a price no more than \( r_j^i \); in other words, she stops searching when the payoff she can get
from firms already searched in channel \( j \) is no less than \( q_j v - r_j^i \) (regardless of search costs
already incurred since they are sunk). She can obtain the same payoff – and thus should also stop
considering channel \( j \) – if she finds a price no more than \( r_i^j \) in channel \( i \).

For this subsection only, assume \( \hat{p}_a > \max\{r_a^a, r_b^b\} \) so that searching either channel is a
better choice for the consumer than stopping. We will later relax this assumption. Now we are
ready to analyze the channel selection problem. Let \( u_{ab} \) denote a consumer’s expected payoff if
she follows the \( a \)-priority path. We have

\[
u_{ab} = -c_a + \eta_a \int_{p_a} \hat{p}_a (\hat{p}_a - p_a) f_a(p_a) dp_a - [\eta_a (1 - F_a(r_a^b)) + (1 - \eta_a) c_b \\
+ [\eta_a (1 - F_a(\hat{p}_a)) + (1 - \eta_a) \int_{p_b} ((q_b v - p_b) - (q_a v - \hat{p}_a)) f_b(p_b) dp_b \\
+ \eta_a \int_{p_a} \eta_b \int_{p_b} ((q_b v - p_b) - (q_a v - p_a)) f_b(p_b) f_a(p_a) dp_a dp_b].
\]

The first item on the right side of equation (5) is the search cost in channel \( a \). Note that
the lowest price the consumer already found in this channel is \( \hat{p}_a \), the second item thus
represents the expected gain (i.e. price reduction) from searching one more firm in channel \( a \) (i.e.
\( a \)-firm). After \( a \)-firm is searched, the consumer should continue searching \( b \)-firm if \( a \)-firm
charges a price higher than \( r_a^b \) or if \( a \)-firm does not carry this product at all – the according search cost is represented by the third item in equation (5). The final two items are the expected gain from searching \( b \)-firm: the former is when the price found at \( a \)-firm is higher than \( \hat{p}_a \), and the latter is when the price found at \( a \)-firm is between \( \hat{p}_a \) and \( r_a^b \).

The consumer’s expected payoff following the \( b \)-priority path, \( u_{ba} \), is analogous with one caveat: since \( q_a v - \hat{p}_a \geq q_b v - \hat{p}_b \), the found best deal is in channel \( a \), and thus \( \hat{p}_b \) is irrelevant to her decision problem. We have

\[
-u_{ba} = -c_b + \eta_b \int_{\mathcal{P}_b} \left( (q_b v - p_b) - (q_a v - \hat{p}_a) \right) f_b(p_b) dp_b - \left[ \eta_b (1 - F_b(r_b^a)) + (1 - \eta_b) \right] c_a ^{\hat{p}_b} \left[ \eta_b (1 - F_b((q_b - q_a)v + \hat{p}_a)) + (1 - \eta_b) \right] \eta_a \int_{\mathcal{P}_a} (\hat{p}_a - p_a) f_a(p_a) dp_a \\
+ \eta_b \int_{\mathcal{P}_b} \eta_a \int_{\mathcal{P}_a} ((q_a v - p_a) - (q_b v - p_b)) f_a(p_a) dp_a f_b(p_b) dp_b.
\]

Both \( u_{ab} \) and \( u_{ba} \) depend on the best found deal, \( q_a v - \hat{p}_a \), which in turn depends on \( \hat{p}_a \).

Therefore, when choosing between continuing or stopping search, a consumer’s optimal decision is history dependent. When choosing which channel to search first (conditional on searching is better than stopping), nevertheless, the next proposition shows a strikingly different result.

**Proposition 2:** Given that only one firm remains unsearched in each channel,

\[
u_{ab} - u_{ba} = \eta_a \eta_b \left\{ -F_b(r^a_b) c_a / \eta_a + F_a(r^b_a) c_b / \eta_b \right\} + \int_\Omega \left\{ ((q_a v - p_a) - (q_b v - p_b)) f_b(p_b) dp_b f_a(p_a) dp_a \right\},
\]

where region \( \Omega \) is shown in Figure 2.
Proposition 2 is significant in that equation (7) does not contain \( \hat{\alpha}_a \) (or \( \hat{\beta}_b \)). In other words, between \( a \)- and \( b \)-priority paths, which one is better does not depend on already found prices. This dramatically reduces the complexity of the consumer search problem and, as we will see shortly, enables us to characterize a consumer’s optimal decisions in the full search model.

Recall that we assumed \( q_a v - \hat{\alpha}_a \geq q_b v - \hat{\beta}_b \) earlier to facilitate the analysis. It turns out Proposition 2 still holds if \( q_a v - \hat{\alpha}_a \leq q_b v - \hat{\beta}_b \) – not surprising since neither \( \hat{\alpha}_a \) nor \( \hat{\beta}_b \) appears in equation (7).

Equation (7) shows that the difference between a consumer’s expected gains from searching \( a \)- and \( b \)-priority paths equals to an expression with many parameters – thus calculating this difference can be complicated. Nevertheless, the next proposition shows that it is relatively easy to determine the sign of this difference.
Proposition 3: Suppose that a consumer has searched all but one firm in each channel. If \( \hat{p}_a \leq r_a^a \) and \( \hat{p}_b \leq r_b^b \), the consumer stops search. Otherwise,

(i) if \( q_a v - r_a^a \geq q_b v - r_b^b \), the consumer follows the a-priority path;

(ii) if \( q_a v - r_a^a < q_b v - r_b^b \), the consumer follows the b-priority path.

3.2. The Full Search Problem

In this subsection we analyze the full search problem, where a consumer faces A firms in channel \( a \) and B firms in channel \( b \), and that the consumer has not searched any. Both A and B are integers larger than one. Recall that, following the sequential search literature (Baye et al. 2006), we assume consumer valuations are high enough so they will search at least one firm.\(^3\)

Given A and B and noting that all firms in the same channel are ex ante homogenous, a consumer faces \( \binom{A+B}{A} \) potential paths. Among these paths, we now call the path that starts with consecutive searches of all firms in channel \( a \) the a-priority path, and the path that starts with consecutive searches of all firms in channel \( b \) the b-priority path. See the figure below for an illustration where each channel contains two firms.

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\(^3\) Nevertheless, the results in this section can be extended to show that, if a consumer’s valuation is low enough (i.e. in every channel lower than the channel’s reservation price), the consumer will not search at all.
In Figure 3, there are 6 paths: $aabb$, $abab$, $abba$, $baab$, $baba$ and $bbaa$. Note that, along each path, a consumer’s search sequence may take the whole path, or most likely take only an earlier portion of this path if she stops. The top/right most path, $aabb$, is the $a$-priority path (denoted by the bold solid lines); the left/bottom most path, $bbaa$, is the $b$-priority path (denoted by the bold dashed lines).

**Proposition 4:** Let $i, j \in \{a,b\}$ and $i \neq j$. For any given consumer, if

$$q_i v - r_i^i \geq q_j v - r_j^j,$$

the consumer searches along the $i$-priority path. She stops and purchases upon finding a price no higher than $r_i^i$ at a channel $i$ firm, or upon finding a price no higher than $r_j^j$ at a channel $j$ firm.

Even though the number of feasible paths grows exponentially in the number of firms, Proposition 4 shows that a consumer’s optimal choice is always one of two candidate paths: the $a$-priority path and the $b$-priority path. One direct implication is that, if $q_a v - r_a^a \neq q_b v - r_b^b$, a
consumer never conducts interleaved search: it is never possible for a consumer to switch from channel \(i\) to \(j\), then switch back to channel \(i\).

We should note that this result of no interleaved search depends on the assumption that consumers know \(v\) and \(\{q_i, q_j\}\) before searching. Previous research has suggested that, if consumers do not know \(v\), and if \(q_i\) and \(q_j\) are far apart, it is possible for a consumer to first sample a low quality firm for information gathering, and then switch to a high quality firm if \(v\) turns out to be large (Chellappa and Shivendu 2005).

The condition for a consumer’s channel decision, (8), is strikingly simple and has an intuitive explanation: \(q_i v - r_i^v\) (\(q_j v - r_j^v\)) is a consumer’s payoff in channel \(i\) (\(j\)) if she finds a price that equals to the reservation price in this channel; and she should pick the channel that offers the best value at the respective reservation price. Note that, if the price distribution in a channel is atomless, the chance for a consumer to stop at a price quote that exactly equals to the reservation price is zero. Instead, she has probability one to stop at a price lower than the reservation price, and how low the found price is depends on the price distribution inside the channel. A strong implication of Proposition 4 is that, in expectation, the detailed shapes of price distributions under the reservation prices are not important for consumers’ decisions.

4. Application to Competition between Legitimate and Piracy Channels

In this section we narrow the scope of the multi-channel search model down to the case of legitimate and piracy channels, where the legitimate channel consists of multiple retailers and the piracy channel consists of multiple piracy services. We first introduce the model specifications used in this section. We then study that, given a price distribution among legitimate retailers,
whether various piracy controls are effective at reducing piracy. Last we analyze the impact of piracy on the pricing decisions of legitimate retailers.

4.1. The Case of Legitimate and Piracy Channels

We use subscripts “l” and “ρ” to denote the legitimate and piracy channels, respectively. We still use A and B to represent the number of retailers and piracy services, respectively. Suppose that product availability is not an issue, i.e. \( \eta_l = \eta_\rho = 1 \). This is case, for instance, when the product is a popular item such as a popular movie. We normalize the quality of the legitimate product to 1. The piracy product’s quality is \( q_\rho = q < 1 \).

As Diamond (1971) pointed out, a sequential search model where all consumers have positive search costs can result in all firms adopting monopoly pricing, which does not fit business practices where price dispersion is widely observed (Baye et al. 2006). Scholars have subsequently proposed various modifications to the Diamond model to address this discrepancy between theory and practice. Among them, one of the most influential works is Stahl (1989), which we follow in this section.

Following Stahl (1989), let there be two types of consumers. A proportion \( \mu \in [0,1] \) of consumers – called *shoppers* – have zero search costs. This is the case, for instance, if a consumer has no opportunity cost of time, or enjoys window shopping, or possesses technologies (such as ShopSavvy on T-Mobile G1) that dramatically reduce search costs. The rest \( 1 - \mu \) consumers – called *non-shoppers* – have positive search costs \( c_l (c_\rho) \) in the legitimate (piracy) channel. Note that the analysis in Section 3 applies to non-shoppers only.
Nowadays most popular piracy services use peer-to-peer technologies such as the BitTorrent protocol, where few have pricing/charging capability and thus piracy products are offered for free. Accordingly, let \( p_\rho \equiv 0 \). Let \( p_i = p \) and let \( F(p) \) denote the price distribution in the legitimate channel.

### 4.2. Piracy Controls when Price Distribution for Legitimate Retailers Fixed

Legitimate retailers may compete with pirates in two broad ways. They can either keep their own prices constant while trying to make the piracy option less attractive to consumers, or they can adjust their prices to adapt to piracy services. We discuss the former in this subsection and the latter in the next one.

Since the piracy channel always offers a zero price, condition (8) do not readily apply. We can rewrite (8) as \((1 - q)v + r_\rho \geq r_i\), or \( r_\rho \geq r_i \) (see Proposition 1). It is straightforward that \( r_\rho = (1 - q)v + c_\rho \). Hereafter we denote \( r_i = r \) for notational convenience. Then, non-shoppers will choose the legitimate channel when \( r \leq (1 - q)v + c_\rho \). Let \( p_{\text{min}} \) denote the lowest price among all A retailers. Shoppers will buy from the retailer offering price \( p_{\text{min}} \) when \( p_{\text{min}} \leq (1 - q)v \). Since neither shoppers nor non-shoppers’ channel choices are dependent on B, we have:

**Proposition 5:** Given \( F(p) \) and that there is at least one piracy service, reducing the number of piracy services does not affect consumers’ channel choices or retailers’ demands.
Proposition 5 casts doubt on the effectiveness of chasing after and shutting down piracy services for reducing piracy. Unless all piracy services are shut down, Proposition 5 says a consumer will not change her piracy choice if retailers maintain a constant pricing strategy.\(^4\)

Proposition 5 is based on the assumption that \( \eta_\rho = 1 \), i.e. piracy services always carry the product. If \( \eta_\rho < 1 \) instead, there is a chance of \((1 - \eta_\rho)^B\) that none of the piracy services carry the product, and thus a consumer that prefers the piracy channel will eventually buy from a legitimate retailer. Nevertheless, when \( B \) is large – which is often the case in practice, \((1 - \eta_\rho)^B\) decreases quickly to near zero and thus the return from shutting down a few piracy services is still very marginal.

### 4.3. The Impact of Piracy on Retailer Pricing

We first analyze the benchmark case where \( B = 0 \), i.e. there is no piracy threat and thus retailers are competing only with each others. Suppose the equilibrium reservation price is \( r_0 \), then apparently no retailer will charge a price higher than \( r_0 \).

If a retailer charges \( p = r_0 \), no shopper will purchase, and thus its profit is

\[
\pi(r_0) = \frac{1 - \mu}{A} r_0.
\]

If it charges \( p < r_0 \), shoppers will purchase if \( p \) is the lowest price among all \( A \) retailers, and thus its profit is

\[
\pi(p) = \left[ \frac{1 - \mu}{A} + \mu (1 - F(p))^{A-1} \right] p.
\]

From \( \pi(r_0) = \pi(p) \) we have:

\[
p = \frac{1}{1 + A(1 - F(p))^{A-1} \mu / (1 - \mu) r_0}.
\]

\[^4\text{We will shortly show in the next subsection that retailer pricing does remain constant when the number of piracy services change.}\]
Therefore, \[ E(p) = \int_0^1 pdF(p) = \int_0^1 \frac{1}{1 + A(1-x)^{A-1} \mu / (1-\mu)} r_0 dx. \]

Also from (3) we have \[ r_0 - E(p) = c_i. \] From the last two equations we get

\[ r_0 = \frac{c_i}{1 - \int_0^1 \frac{1}{1 + A(1-x)^{A-1} \mu / (1-\mu)} dx}. \] (10)

Therefore, \( r_0 \) increases in \( c_i \) and \( A \), and decreases in \( \mu \). We next analyze the case where \( B > 1 \).

**Proposition 6:** If \( r_0 \leq (1-q)v \), \( r = r_0 \) and the piracy threat does not affect retailer demand and pricing.

Intuitively, when \( r_0 \leq (1-q)v \), in-channel competition among retailers is already intensive, and as a result no retailer will price above \((1-q)v\) – the quality premium the legitimate channel has over the piracy channel. As a result, neither shoppers nor non-shoppers will consider the piracy product, which in turn implies that retailer demands are not affected by piracy. As shown in equation (10), multiple factors may lead to intensive in-channel competition among retailers: a low search cost in the legitimate channel, or is small number of retailers, or a higher proportion of shoppers.

On the other hand, when \( r_0 > (1-q)v \), the piracy threat will affect decisions by retailers as now consumers may find the piracy product attractive. Let \( \theta(r) = \left[ \frac{1-\mu}{A\mu} \left( \frac{r}{(1-q)v} - 1 \right) \right]^{\frac{1}{A-1}} \) and let \( r^* \) be the solution to \( r^* [1-\theta(r^*)] - \int_0^{1-\theta(r^*)} \frac{1}{1 + A(1-x)^{A-1} \mu / (1-\mu)} dx = c_i \). Since \( \theta(r) \) is monotonically increasing, it is straightforward that \( r^* > r_0 \).
Proposition 7: If \( r_v > (1-q)v \) and \( r^* \leq (1-q)v + c_\rho \), \( r = r^* \) and

\[
F(p) = \begin{cases} 
1 & \text{if } p = r^* \\
1 - \theta(r^*) & \text{if } (1-q)v \leq p < r^* \\
1 - \left[ \frac{r^* - p \cdot \frac{1-\mu}{\lambda - 1}}{p} \right]^{\frac{1}{\lambda - 1}} & \text{if } p \leq (1-q)v
\end{cases}
\]

Proposition 7 has a number of interesting implications. First, piracy threat increases the expected price charged by legitimate retailers (recall that \( E(p) = r^* - c_i \)). This is surprising as one might expect cross-channel competition to put downward pressure on prices. To understand this result, consider the two types of competition a retailer faces: one from piracy services and the other from peer retailers. Without piracy services, the retailer will compete with in-channel peers for both non-shoppers and shoppers – the latter type of consumers lead to intensive price competition among retailers. With the piracy threat, however, a retailer that charges \( p = r^* \) gives up shoppers completely, and thus is competing with in-channel peers only for non-shoppers, on whom the retailer can charge a higher price. To summarize, cross-channel competition actually reduces in-channel competition among retailers, thus leading to a higher reservation price and a higher expected price.

Second, the piracy threat leads to polarized pricing among retailers. Note that \( \theta(r^*) \) proportion of retailers charge the highest possible price – the reservation price. At the meantime, no retailer charges any price within \((1-q)v, r^*\). Intuitively, if a retailer charges a price within
\((1-q)v, r\)\), only non-shoppers will purchase. But then this retailer can increase its price to \(r^*\) without losing any demand.

Piracy threats can drive up retailer prices, but only to a limited extent. Note that, if the reservation in the legitimate channel is higher than \((1-q)v + c_\rho\), no consumer will purchase from this retailer. Therefore, \((1-q)v + c_\rho\) is an upper ceiling for \(r\). Let 
\[
\tilde{\theta}(\gamma) = \left[\frac{1 - \mu}{A\mu} \cdot \frac{c_\rho}{(1-q)v} \gamma \right]^{1/(A-1)}
\]
and let \(\tilde{\gamma}\) be the solution to 
\[
((1-q)v + c_\rho)[1 - \tilde{\theta}(\tilde{\gamma}) - \int_{0}^{1-\tilde{\theta}(\tilde{\gamma})} \frac{1}{1 + A(1-x)^{A-1}} \mu \frac{1}{((1-\mu)\tilde{\gamma})} dx] = c_i.
\]

**Proposition 8:** If \(r^* > (1-q)v + c_\rho\), \(r = (1-q)v + c_\rho\) and

\[
F(p) = \begin{cases} 
1 & \text{if } p = (1-q)v + c_\rho \\
1 - \tilde{\theta}(\tilde{\gamma}) & \text{if } (1-q)v \leq p < (1-q)v + c_\rho \\
1 - \left[\frac{(1-q)v + c_\rho - p}{p} \cdot \frac{\tilde{\gamma}(1-\mu)}{A\mu} \right]^{1/(A-1)} & \text{if } p \leq (1-q)v
\end{cases}
\]

Moreover, non-shoppers select the legitimate channel with probability \(\tilde{\gamma}\).

Proposition 8 is analogous to Proposition 7 in that retailer prices are also polarized. Nevertheless, under Proposition 8 retailers are losing not only some shoppers, but also some non-shoppers since \(\tilde{\gamma} < 1\).

Proposition 7 and 8 together shows that, when the competition from the piracy channel is strong, retailers may give up low cost consumers, which in turn reduces the in-channel competition among retailers. As a result, piracy does not reduce the price retailers charge. This
result is consistent with the fact that, despite years of piracy threats, major music labels have not significantly reduced CD prices.

5. Concluding Remarks

This paper studies a multi-channel search model in which channels may differ in quality, product availability, search costs and price distributions. We show that a consumer’s channel choice at any stage of the search is uniquely decided by every channel’s reservation price and consumer value, and is independent of found price quotes or the number of firms left.

We then apply this model to analyzing the competition between legitimate products and piracy products. Our analysis yields a number of striking results. First, shutting down piracy services (except shutting down all) does not benefit legitimate retailers. Second, where piracy affects pricing by legitimate retailers depends on the in-channel competiveness among retailers. If in-channel competition is already intense enough, legitimate retailers will be charging low prices, and thus piracy services do not affect the demand of legitimate products. If in-channel competition is not intense enough, the threat of piracy may force some retailers to give up low search cost consumers, which actually reduces in-channel competition among retailers. As a result, legitimate retailers may increase prices in the face of piracy threats.

This research lays a foundation for future research on multi-channel search issues. One possibly is to apply the framework to other types of cross-channel competitions, such as competition between online and offline channels. Another possibly is to introduce valuation uncertainty into the model, so that a consumer’s search process is not only a price discovery process, but also a learning process with respect to product valuations.
Appendix

Proof of Proposition 2:

We only prove the case where \( r_a^a \geq r_b^b \) (the case of \( r_a^a < r_b^b \) is analogous). For notational convenience, we use \( u_{ab}[k] \) to denote the \( k'th \) component (sign included) on the right of size of equation (5).

\[
u_{ab}[1] - u_{ba}[3] = -\eta_b F_b(r_b^b)c_a. \]

\[
u_{ab}[3] - u_{ba}[1] = \eta_a F_a(r_a^a)c_b. \]

\[
u_{ab}[2] - u_{ba}[4] = \eta_a \eta_b F_b((q_b - q_a)v + \hat{p}_a) \int_{\hat{p}_b}^{\hat{p}_a} (\hat{p}_a - p_a)f_a(p_a)dp_a. \]

\[
u_{ab}[4] - u_{ba}[2] = -\eta_a \eta_b F_a(\hat{p}_a) \int_{\hat{p}_b}^{(q_b - q_a)v + \hat{p}_a} ((q_b v - p_b) - (q_a v - \hat{p}_a))f_b(p_b)dp_b. \]

\[
u_{ab}[5] - u_{ba}[5] = (\int - \int_\Omega |p_a| \leq p_a \leq \hat{p}_a \int_{\hat{p}_b}^{q_b - q_a v + \hat{p}_a} ((q_a v - p_a) - (q_b v - p_b))f_b(p_b)dp_b f_a(p_a)dp_a. \]

Then we can verify that \((u_{ab}[5] - u_{ba}[5]) + (u_{ab}[2] - u_{ba}[4]) + (u_{ab}[4] - u_{ba}[2])\)

\[
\Omega \int ((q_a v - p_a) - (q_b v - p_b))f_b(p_b)dp_b f_a(p_a)dp_a. \quad \text{Q.E.D.}
\]

Proof of Proposition 3:

We only consider the case where \( q_a v - r_a^a < q_b v - r_b^b \).

\[-F_b(r_b^b)c_a / \eta_a = -\int_{\hat{p}_b}^{\hat{p}_a} f_b(p_b)dp_b \int_{\hat{p}_b}^{\hat{p}_a} (r_a^a - p_a)f_a(p_a)dp_a. \]
\[
F_a(r^b_a)c_b / \eta_b = \int_{p_a}^{r^a_a} f_a(p_a)dp_a \cdot \int_{p_b}^{r^b_b} (r^b_b - p_b) f_b(p_b)dp_b.
\]

Therefore, equation (7) can be rewritten as
\[
(u_{ab} - u_{ba}) / \eta_a \eta_b = \int_{\Omega_1} (p_b - r^b_b) f_b(p_b)dp_b f_a(p_a)dp_a \\
+ \int_{\Omega_2} ((q_a v - r^a_a) - (q_b v - r^b_b)) f_b(p_b)dp_b f_a(p_a)dp_a \\
+ \int_{\Omega_3} (p_a - r^a_a) f_b(p_b)dp_b f_a(p_a)dp_a,
\]
where areas \( \Omega_1 \), \( \Omega_2 \) and \( \Omega_3 \) are shown in the figure below. All three components in the right side of the above equation are negative.

Proof of Proposition 7:

Because \( r_0 > (1 - q)v \), \( r \leq (1 - q)v \) is impossible: otherwise, from Proposition 6 we will have \( r = r_0 > (1 - q)v \). Hereafter we consider \( r > (1 - q)v \).

Suppose that \( r \leq (1 - q)v + c_p \) (we will verify this later), thus non-shoppers first search the legitimate channel. It is straightforward that no retailer will pick any price between \( (1 - q)v \) and \( r \): otherwise, the retailer can increase its price to \( r \) without losing any sales. A direct result of this observation is that there must exist a mass at \( p > (1 - q)v \); let this mass be \( \theta \).
If a retailer sets \( p = r \), its profit is \( \pi(r) = r \frac{1 - \mu}{A} \). If it sets \( p = (1 - q)v \), its profit is
\[
\pi((1 - q)v) = (1 - q)v \left[ \frac{1 - \mu}{A} + \mu \theta^{\frac{1}{A - 1}} \right].
\]
Equaling the above two profits, we have
\[
\theta = \left[ \frac{1 - \mu}{A \mu (1 - q)v} - 1 \right]^{\frac{1}{A - 1}}. \text{ Note that } \theta \text{ increases in } r.
\]
If the retailer sets \( p < (1 - q)v \), its profit is \( \pi(p) = p \left[ \frac{1 - \mu}{A} + \mu (1 - F(p))^{\frac{1}{A - 1}} \right] \). Thus we have \( p = \frac{r}{1 + A(1 - F(p))^{\frac{1}{A - 1}} \mu / (1 - \mu)} \). Then,
\[
E(p) = \theta r + \int_{\gamma}^{(1-q)v} \frac{r}{1 + A(1 - F(p))^{\frac{1}{A - 1}} \mu / (1 - \mu)} dF(p) = \theta r + r \int_{0}^{\theta} \frac{1}{1 + A(1 - x)^{\frac{1}{A - 1}} \mu / (1 - \mu)} dx.
\]
Also recall that \( E(p) = r - c_i \). \( r^* \) is the unique reservation price that satisfies the last two equations simultaneously. From \( \pi(p) = \pi(r^*) \) we can derive \( F(p) \) when \( p \leq (1 - q)v \). Q.E.D.

**Proof of Proposition 8:**

First note that \( r > (1 - q)v + c_\rho \) is impossible: otherwise, a retailer charging \( p = r \) gets zero profit. \( r < (1 - q)v + c_\rho \) is also impossible: otherwise, \( r = r^* \) which contradicts the assumption that \( r^* > (1 - q)v + c_\rho \). Therefore, it must be true that \( r = (1 - q)v + c_\rho \).

At \( p = r = (1 - q)v + c_\rho \), a non-shopper is indifferent between the two channels. Let \( \gamma \) proportion of non-shoppers choose the legitimate channel when \( p = (1 - q)v + c_\rho \). Then a retailer’s profit at \( p = (1 - q)v + c_\rho \) is \( \pi((1 - q)v + c_\rho) = ((1 - q)v + c_\rho) \frac{\gamma(1 - \mu)}{A} \). If it sets
\( p = (1-q)v \), its profit is \( \pi((1-q)v) = (1-q)v \left[ \frac{\gamma(1-\mu)}{A} + \mu\tilde{\theta}^{\gamma-1} \right] \), where \( \tilde{\theta} \) is the mass of retailers at this price. Equaling the above two profits, we have \( \tilde{\theta} = \frac{\gamma(1-\mu) \cdot c_{\rho}}{A\mu} \frac{1}{(1-q)v} \). Note that \( \tilde{\theta} \) increases in \( \gamma \).

If the retailer sets \( p < (1-q)v \), its profit is \( \pi(p) = p\left[ \frac{\gamma(1-\mu)}{A} + \mu(1-F(p))^{\gamma-1} \right] \). Thus we have \( p = \frac{(1-q)v + c_{\rho}}{1 + A(1-F(p))^{\gamma-1} \mu / ((1-\mu)\gamma)} \). Then,

\[
E(p) = \tilde{\theta}((1-q)v + c_{\rho}) + \int_0^{1-\tilde{\theta}} \frac{(1-q)v + c_{\rho}}{1 + A(1-x)^{\gamma-1} \mu / ((1-\mu)\gamma)} \, dx.
\]

Also recall that \( E(p) = r - c_{\gamma} \). \( \tilde{\gamma} \) is the unique value of \( \gamma \) that satisfies the last two equations simultaneously. From \( \pi(p) = \pi((1-q)v + c_{\rho}) \) we can derive \( F(p) \) when \( p \leq (1-q)v \). Q.E.D.

References


