Use of Pricing Schemes for Differentiating Information Goods

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Abstract

Information goods vendors offer many different pricing schemes such as per user pricing, site licensing, etc. Why do some firms such as Microsoft offer multiple pricing schemes while others such as Apple’s iTunes and Salesforce.com limit themselves to a single pricing scheme? Why do competing sellers adopt different pricing schemes for the same information good? We show that pricing schemes affect buyers’ usage levels and thus the revenue generated from them. This is reflected in differences in buyers’ demand elasticity under different pricing schemes. We propose and formalize a concept we call Congruousness to measure the level of fit between pricing schemes and buyer segments. We show that firms can use pricing schemes with different congruousness to differentiate themselves from competing firms in a friction-free market for a commoditized information good. Under conditions that are known to result in the Bertrand equilibrium, we show that an undifferentiated duopoly of sellers can earn substantial profits by using different pricing schemes in a strategic manner. Contrary to prior literature, we show that the strategy of adopting asymmetric pricing schemes can be a Nash equilibrium for information goods with negligible marginal costs of production. We extend our model to the case of information goods that are horizontally differentiated and show that sellers will offer a single pricing scheme that is different from its competitor when the sellers are weakly differentiated. When the sellers are strongly differentiated, each seller will offer multiple pricing schemes. We show that it can be optimal for a seller to offer multiple pricing schemes – metered and flat fee pricing schemes even in the absence of transactions costs.

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1 Introduction

Information goods have become a large and growing part of the United States economy. Information goods are commonly defined as products that can be digitized such as books, software, videos, music, telecommunications services etc. Vendors of information goods offer many different pricing schemes. For example Microsoft offers a number of pricing schemes such as Open License, Select License, Enterprise Agreement, Enterprise Subscription Agreement, etc. On the other hand Salesforce.com, a leading provider of CRM software offers only one pricing scheme – subscription based pricing. Similarly Apple’s iTunes music download service offers only one pricing scheme – a fixed price for each song. In contrast, Yahoo’s competing music download service also offers only one pricing scheme – subscription service with a monthly fee for unlimited downloads. How can we explain these contrasting outcomes where some firms offer multiple pricing schemes and others offer only one pricing scheme but choose a pricing scheme that is different from their competitor? In this paper we examine this heterogeneity in pricing schemes and offer an explanation for this observed phenomenon. We posit that certain pricing schemes fit certain buyers’ usage patterns. For example, the ‘family pricing plans’ offered by cell phone providers are designed to appeal to the usage patterns of families with a particular mix of metered pricing for some calls and unlimited usage for calls to family members. Offering pricing schemes to appeal to different buyer segments is similar to enhancing particular product features to appeal to different buyer segments with some differences. We will show that the use of different pricing schemes by undifferentiated or weakly differentiated sellers can enhance differentiation among competing sellers. Sellers that are well differentiated find it optimal to offer multiple pricing schemes. These findings support the commonly observed phenomenon of multiple pricing schemes offered by information goods vendors such as cell phone providers, software publishers and music download services.

To better understand the role of pricing schemes and differentiation, note that sellers prefer to be differentiated from their competitors since enhancing differentiation helps to alleviate price competition and increases profit. Our finding that pricing schemes can be used to enhance differentiation is the aggregation of a number of smaller steps. We assume that buyers have utility for
multiple units of the information good. A firm may buy a number of single-user software licenses for their employees or an individual may have utility for multiple units of ‘minutes’ for cell phone calling. Buyers are assumed to be heterogeneous in their utility for the information good, both for the first unit as well as subsequent units. Thus some buyers may have a relatively high utility for the first unit – say a senior citizen who uses the cell phone in case of emergencies while others may have a relatively higher utility from consuming multiple units such as a teenager with friends or a travelling salesman. The first important effect of different pricing schemes is that they affect the buyer’s consumption level in terms of the number of units consumed. For example a rational cell phone user is likely to consume fewer minutes when using a calling plan that charges by the minute vs. using a calling plan with a fixed price for unlimited calling. In a corporate environment, managers are likely to encourage wider adoption of software when it is purchased as a site license compared to when the manager has to pay for each additional license. The second issue to note is that the heterogeneity in the marginal utility of different consumers implies that there exist a set of prices for the metered pricing scheme and the unlimited usage scheme such that some consumers prefer the metered pricing scheme while others prefer the unlimited consumption scheme. For example at typical prices, cell phone users who expect limited usage in emergencies prefer a per minute calling plan whereas others prefer calling plans with large or unlimited minutes. Similarly small businesses often buy single user licenses while large corporations routinely purchase enterprise (site) licenses. Thirdly, note that when vendors are undifferentiated or weakly differentiated from their competitors, they face intense price competition and earn relatively small profits (relative to that of a monopoly). Combining the three previous observations, we can see that when one seller offers metered pricing while the other charges a flat fee for unlimited consumption, that buyers will prefer different vendors depending on their marginal utility. Thus the two sellers can enhance their differentiation based on their choice of different pricing schemes. When the sellers are already well differentiated they choose to offer multiple pricing schemes. In either case such markets will exhibit a variety of pricing schemes.
1.1 Literature

There is substantial research on pricing in the Information Systems literature. Dewan and Mendelson [1990], Mendelson [1985], Mendelson and Whang [1990] examine optimal pricing policies in the presence of user delay costs. Westland [1992] examines information systems pricing in the presence of network externalities. Terwiesch, Savin, and Hann [2005] and Hann and Terwiesch [2003] study haggling and frictional costs at a Name Your Own Price online retailer. In the context of bundling as a pricing strategy, Bakos and Brynjolfsson [1999, 2002] show that bundling large numbers of unrelated information goods can be very profitable whereas the same is not true for physical goods. Geng, Stinchcombe and Whinston [2005] show that bundling is suboptimal when users marginal value of additional information decreases quickly. Hitt and Chen [2005] examine the pricing strategy of customized bundling and show that it can be more profitable than pure bundling when the goods have low marginal costs.

Most pricing models can be classified as either vertical models where all buyers agree on a quality based rank-ordering of the product offerings or horizontal models where buyers have heterogeneous preferences among products even when they are offered at the same price. Maskin and Riley [1984] develop a vertical model to study the strategy of a multi-product monopolist while Moorthy [1988] examines duopolistic competition in a vertically differentiated framework. Bhargava and Choudhary [2001] examines versioning of information goods and establishes conditions under which a monopolist should offer a single quality. Sundararajan [2004a] models digital pirated copies as being vertically differentiated (lower quality) relative to the legal good whereas Chellappa and Shivendu’s [2005] model of piracy incorporates both horizontal and vertical elements. Other recent papers that have used vertical models in information systems context include Bakos et al.’s [2005] analysis of the retail brokerage industry, Bhargava and Choudhary’s [2004] analysis of pricing strategies for an electronic intermediary, Riggins [2003] analysis of the trade-off between subscription fees and advertising revenue for an informational website, and Thatcher and Pingry’s [2004] analysis of value from IT investments.

Horizontal differentiation models are based on the work of Hotelling [1929] and refined by
d’Aspremont, Gabszewicz, and Thisse [1979] who proposed the principle of maximal differentiation. Whereas vertical models assume that consumers agree on a rank ordering of competing products based on their quality; horizontal models capture the fit between users with different requirements and products that may fit some users’ requirements better than others. For example, comparing the New York Times to the Wall Street Journal, it is likely that users who are interested in politics and culture may prefer the New York Times while financial analysts and investors are likely to prefer the Wall Street Journal. A bank that is looking to buy database software, is likely to require strong software security whereas an arts studio may focus on ease of installation and use. Thus different users have different requirements and products may better meet the needs of certain segments of users. Horizontal differentiation models have been used widely in Information systems, Marketing and Economics literature. Dewan, Jing and Seidmann [2003] use horizontal models to examine online product customization. They find that while product customization reduces differentiation, it does not aggravate price competition for the standard product. Dewan et al. [2004] study media concentration among web sites that are horizontally differentiated. Telang, Rajan and Mukhopadhyay [2004] study horizontal differentiation among Internet search engines. Viswanathan [2005] uses a horizontal model to examine competition between firms selling across differentiated channels in the presence of network effects and switching costs.

Several papers have examined fixed fee and usage based pricing schemes. Danaher [2002] analyzes data from a field experiment of mobile phone subscribers who are offered contracts with different access (fixed monthly) fee and usage fees. They estimate price elasticities for the usage and access fees and report different price sensitivity for usage and access fee in their two segment model. Jain and Kannan [2002] analyze pricing of a library information service and show that the two sellers may offer different pricing schemes to exploit consumer uncertainty about demand but only when marginal costs are strictly positive. Masuda and Whang [2006] examine the performance of ‘Fixed up-to’ (FUT) plans and show that a monopolist earns at least as much profit with a simple FUT menu plan as with any other nonlinear pricing scheme. Sundararajan [2004b] shows that a monopolist information goods provider will benefit from providing fixed fee pricing in addition to a
non-linear usage based pricing scheme in the presence of non-zero transactions costs. Among other pricing strategies, Desai and Purohit [2004] identify potential equilibria when a retailer can offer a fixed pricing scheme or choose to haggle over prices. In their model haggling imposes costs on both buyers and sellers and more price sensitive buyers incur lower opportunity costs of haggling. Chellappa, Sin and Siddarth [2005] examine data from the domestic US airline market finding support for their hypothesis that price-formats are an important source of price dispersion.

In this paper, we show that sellers can use different pricing schemes to take advantage of heterogeneity in buyer demand for multiple units of an information good. We define two measures of congruousness that formalizes the notion of ‘fit’ between buyers’ demand patterns and pricing schemes. Differences in congruousness across buyers arise from heterogeneity in buyer demand for the underlying information good (buyers need not have any intrinsic preference for any pricing scheme). We demonstrate that the adoption of different pricing schemes by weakly differentiated sellers is a Pareto dominant Nash equilibrium. Contrary to prior literature (Jain and Kannan [2002]), we show that such a strategy can be optimal for information goods with negligible marginal costs of production. We show that pricing schemes can help sellers avoid the Bertrand equilibrium when they are completely undifferentiated. We also show that it is optimal for sellers to offer multiple pricing schemes when they are well differentiated even in the absence of transactions costs. While there are similarities between pricing schemes and product attributes that are used for horizontal differentiation, we find that the principle of maximal differentiation which applies to product attributes does not always apply to pricing schemes.

We begin in §2 with a model of duopolistic sellers selling a commoditized information good. We extend this to a model with horizontally differentiated sellers in §3. We show that in markets where the information goods are highly differentiated, each firm will offer both pricing schemes but in highly competitive markets, firms will use different pricing schemes. We discuss our findings and their relationship to prior literature in §4 and conclude in §5.
2 The Case of Commoditized Information Goods

Our analysis focusses on two commonly observed pricing schemes: site licensing or unlimited usage pricing and per unit pricing or metered usage pricing. These pricing schemes are commonly observed and have been studied in the literature. To improve exposition, we will discuss our model in the context of buyers as firms that want to purchase software licences for some or all of their employees. The site pricing scheme allows an unlimited number of users to access the purchased software while the unit pricing scheme charges for each user.

We model two undifferentiated sellers with commoditized (perfectly substitutable) information goods with no marginal costs of production. Buyer are firms with multiple employees that could potentially benefit from the software product. Buyers are perfectly informed about prices and there are no transactions costs. There exist two broad classes of buyers, one with a declining marginal willingness to pay (D-type) for multiple units and the second with a constant marginal willingness to pay (C-type) for a fixed number of units. For example a firm may have the usual declining marginal WTP for Adobe Acrobat software when it is used by individual employees for productivity improvements. On the other hand, firms that plan to use Acrobat as part of their enterprise system for process automation and work flow require certain employees to use the software and therefore need a finite number of licenses. This could be the case in other instances where firms adopt a technology as a company standard. Let $k$ fraction of the buyers be D-type and the remaining $(1 - k)$ be C-type buyers. The willingness to pay of each D-type buyer can be expressed in terms of the inverse demand function: $p_u = \alpha - q$ where $p_u$ is the per unit price of the software, $q$ the quantity of licenses bought by each buyer, and $\alpha$ the willingness to pay for the first license. The net surplus for a D-type buyer for $q$ units is:

\[
\text{Unit Pricing: } \left( \int_0^q (\alpha - x)dx \right) - p_u \cdot q \quad \text{Site Licensing: } \left( \int_0^q (\alpha - x)dx \right) - p_s
\]

where $p_s$ is the price for the site license. The surplus from buying a site license is $\frac{\alpha^2}{2} - p_s$. If the site license involves a stream of payments then $p_s$ is the discounted present value of the payment.
stream. The marginal costs of production are assumed to be negligible and the fixed costs are constant. Thus they do not play a role in determining the equilibrium solution as long as the total profit is greater than the fixed cost.

The cumulative WTP for each of the C-type buyers is $\omega \cdot n$ where $\omega$ is the average benefit per user and $n$ the number of users. These buyers yield a discrete demand function where they buy $n$ licenses for a price up to $\omega n$. Across this segment of buyers, let $n$ be uniformly distributed over $[0, 1]$.

### 2.1 Benchmark Models

We begin by solving two benchmark cases, one where a firm offers unit pricing scheme and another where a firm offers site pricing scheme. These results are useful for comparison with the duopoly case. We also use them in the discussion section to illustrate the nature of the aggregate demand function for the C and D buyer types and the difference in their demand elasticity with respect to the two different pricing schemes.

#### 2.1.1 Benchmark: Unit Based Pricing Scheme

We first analyze the case where a firm offers the unit pricing scheme to both C and D type buyers. The relative proportion of D-type buyers is $k$ and the remaining $(1 - k)$ buyers are C-type buyers. At a given unit price, depending on the value of the parameters $\alpha$ and $\omega$, there may be demand from both C and D type buyers or from only C-type or from only D-type buyers. The seller’s profit function can be stated as:

\[
\pi_u = \begin{cases} 
  p_u \cdot k(\alpha - p_u) + p_u \frac{(1-k)}{2} & \text{when } p_u \leq \omega \text{ & } p_u \leq \alpha \\
  p_u \cdot k(\alpha - p_u) & \text{when } \omega < p_u \leq \alpha \\
  p_u \frac{(1-k)}{2} & \text{when } \alpha < p_u \leq \omega 
\end{cases}
\]  

Figure 1 shows the two segments of the profit function when $\omega > \alpha$. The curved portion represents the interior feasible region where prices are low enough to attract both D and C-type
Figure 1: Plot of profit from using unit pricing scheme \((\omega = 24, \alpha = 10, k = 0.25)\).

buyers. The upward sloping straight line \((p_u > 10)\) represents profits from only the C-type buyers. To determine optimal profit, we must compare the profit at the interior maxima to the profit at the boundary. Depending on the parameter values \((\alpha, \omega)\) there are different boundary solutions that need to be considered.

Consider first the region where \(p_u \leq \omega\) and \(p_u \leq \alpha\). The profit is \(\pi_u = p_u \cdot k(\alpha - p_u) + p_u \cdot \frac{(1-k)}{2}\).

Solving \(\frac{\partial \pi_u}{\partial p_u} = 0\), we obtain the following candidate maxima (second order conditions are satisfied):

\[
\hat{p}_u = \frac{\alpha}{2} + \frac{1 - k}{4k}
\] (2)

This solution is feasible when \(\hat{p}_u \leq \alpha\) and \(\hat{p}_u \leq \omega\). These conditions are satisfied when \(k \geq \frac{1}{1+2\alpha}\) and \(k \geq \frac{1}{1-2\alpha+4\omega}\). When these conditions are satisfied, the optimal price can be determined by comparing the profit at \(\hat{p}_u\) to the profit at the boundary \(p_u = \omega\). When \(\hat{p}_u\) is not feasible, the optimal price can be determined by comparing the profit at the boundary \(p_u = \omega\) which maximizes profit from C-type buyers to the profit at \(p_u = \frac{\alpha}{2}\) which maximizes the profit from D-type buyers.

The optimal solution is stated in the proposition below:

**Proposition 1.** The optimal price and profit for a firm selling to C and D type buyers using the unit pricing scheme is

1. \(p_u^* = \hat{p}_u = \frac{\alpha}{2} + \frac{1-k}{4k}\) and \(\pi_u^* = \frac{(1+k(2\alpha-1))^2}{16k}\) (1a) when \(k \geq \frac{1}{1-2\alpha+4\omega}\) and \(\omega \leq \alpha\), or (1b) when \(k \geq \frac{1}{1+2\alpha}\) and \(\omega > \alpha\), and \(\omega \leq \frac{(1+k(2\alpha-1))^2}{8k(1-k)}\)
2. \( p_u^* = \omega \) (2a) when \( k \geq \frac{1}{1+2\alpha} \) and \( \omega > \alpha \), and \( \omega > \frac{(1+k(2\alpha-1))^2}{8k(1-k)} \) then \( \pi_u^* = \frac{(1-k)\omega}{2} \) (2b) when \( k < \frac{1}{1+2\alpha} \) and \( \omega > \alpha \) then \( \pi_u^* = \frac{(1-k)\omega}{2} \) (2c) when \( k < \frac{1}{1+2\alpha} \) and \( \omega \leq \alpha \) and \( k < \frac{2\omega}{2\omega + (\alpha - 2\omega)^2} \).

3. \( p_u^* = \frac{\alpha}{2} \) and \( \pi_u^* = \frac{k\alpha^2}{4} \) when \( k < \frac{1}{1-2\alpha+4\omega} \) and \( k > \frac{2\omega}{2\omega + (\alpha - 2\omega)^2} \).

**Proof.** Please see Appendix A.1 for proof.

The solution obtained in Proposition 1 above serves as a useful benchmark for comparing to the optimal price and profit shown later in Proposition 3. It is useful to note that while in some cases the firm’s optimal price induces participation from both C and D type buyers, in other cases the firm finds it optimal either to sell only to the C-type buyers or only to D-type buyers. We show in §2.3, that a firm that offers the unit pricing scheme obtains a relatively larger proportion of its revenue from C-type buyers compared to D-type buyers, controlling for the aggregate demand from each buyer type. For example, when \( \alpha = \omega = 1 \) and \( k = 1/2 \), then the maximum possible aggregate surplus to all C-type buyers (\( k\alpha^2/2 \)) is equal to that for all D-type buyers (\( (1-k)\omega/2 \)). Applying proposition 1, we find that the firm earns 33\% of its profit from D-type buyers and the remaining 67\% from C-type buyers.

### 2.1.2 Benchmark: Site Licensing Scheme

Now consider a firm selling to both C and D-type buyers using the site pricing scheme. The profit function is

\[
\pi_s = \begin{cases} 
  p_s \cdot k + p_s \cdot \frac{(1-k)\omega-p_s}{\omega} & \text{when } p_s \leq \omega \text{ & } p_s \leq \frac{\alpha^2}{2} \\
  p_s \cdot k & \text{when } \omega < p_s \leq \frac{\alpha^2}{2} \\
  p_s \cdot \frac{(1-k)\omega-p_s}{\omega} & \text{when } \frac{\alpha^2}{2} < p_s \leq \omega 
\end{cases}
\]

(3)

The plot of the profit function is similar to the one in figure 1 and again there is an interior candidate solution and a number of potential boundary solutions. Consider first the region where \( p_s \leq \omega \) and \( p_s \leq \alpha^2/2 \). The profit is \( \pi_s = p_s \cdot k + p_s \cdot \frac{(1-k)\omega-p_s}{\omega} \). Solving \( \frac{\partial \pi_s}{\partial p_s} = 0 \), we obtain the...
following candidate maxima (second order conditions are satisfied):

\[ \hat{p}_s = \frac{\omega}{2(1-k)} \]  

(4)

This solution is feasible when \( \hat{p}_s \leq \alpha^2/2 \) and \( \hat{p}_s \leq \omega \). These conditions are satisfied when \( k \leq \frac{1}{2} \) and \( k \leq 1 - \frac{\omega}{\alpha^2} \). When these conditions are satisfied, the optimal price can be determined by comparing the profit at \( \hat{p}_s \) to the profit at the boundary \( p_s = \alpha^2/2 \). When \( \hat{p}_s \) is not feasible, the optimal price can be determined by comparing the profit at the boundary \( p_s = \omega/2 \) which maximizes profit from C-type buyers to the profit at \( p_s = \alpha^2/2 \), which maximizes the profit from D-type buyers.

**Proposition 2.** The optimal price for a firm selling to C and D type buyers using the site licensing scheme is

1. \( p_s^* = \hat{p}_s = \frac{\omega}{2(1-k)} \) and \( \pi_s^* = \frac{\omega}{4(1-k)} \) (1a) when \( \omega > \alpha^2/2 \) and \( k \leq 1 - \frac{\omega}{\alpha^2} \) or (1b) when \( \omega \leq \alpha^2/2 \) and \( k < 1/2 \) and \( \omega \geq 2k(1-k)\alpha^2 \)

2. \( p_s^* = \alpha^2/2 \) (2a) when \( \omega \leq \alpha^2/2 \) and \( k < 1/2 \) and \( \omega < 2k(1-k)\alpha^2 \) then \( \pi_s^* = \frac{k\alpha^2}{2} \) (2b) when \( \omega \leq \alpha^2/2 \) and \( k > 1/2 \) then \( \pi_s^* = \frac{k\alpha^2}{2} \) (2c) when \( \omega > \alpha^2/2 \) and \( k > 1 - \frac{\omega}{\alpha^2} \) and \( k \leq 1 - \frac{2\alpha^2\omega}{\alpha^4+\omega^2} \) then \( \pi_s^* = \frac{2\alpha^2\omega-(1-k)\alpha^4}{4\omega} \)

3. \( p_s^* = \omega/2 \) and \( \pi_s^* = \frac{(1-k)\omega}{4} \) when \( \omega > \alpha^2/2 \) and \( k > 1 - \frac{\omega}{\alpha^2} \) and \( k > 1 - \frac{2\alpha^2\omega}{\alpha^4+\omega^2} \).

**Proof.** Please see Appendix A.2 for proof. ■

Again we see that the optimal prices may be such that some segments do not purchase at all (parts 2a, 2b and 3 above). Also when \( \alpha = \omega = 1 \) and \( k = 1/2 \), then the maximum possible aggregate surplus to all C-type buyers \( (k\alpha^2/2) \) is equal to that for all D-type buyers \( ((1-k)\omega/2) \). Applying proposition 2, we find that the firm earns 33% of its profit from C-type buyers and the remaining 67% from D-type buyers. Comparing this to the relative profit from the two segments under unit pricing as discussed after proposition 1, we can see that the unit pricing scheme earns a greater share of profits from the C-type buyers while the site pricing scheme earns a greater share...
from the D-type segment. We discuss this issue further in §2.3.

It is also evident from propositions 1 and 2 that a monopolist will find it optimal to offer both pricing schemes under certain conditions. In particular when $\alpha < \omega$ and $\alpha^2/2 > \omega$ then the monopolist will offer both pricing schemes, setting prices $p_s = \alpha^2/2$ and $p_u = \omega$ covering both C and D segments. There exist other conditions such that if either segment holds significantly lower revenue potential relative to the other segment, then the monopolist will offer only one pricing scheme – for example when $\alpha > \omega$ and $\alpha^2/2 > \omega$ and $k \approx 1$ then the monopolist will offer only the site pricing scheme.

2.2 Duopolistic Competition

In this section, we determine the optimal pricing strategy for two firms selling a commoditized information good. Following Desai and Purohit [2004], we model a two-stage game where the sellers determine the pricing scheme in the first stage and their optimal prices in the second stage. In the first stage, each seller selects from three options among pricing schemes: (i) unit pricing scheme, (ii) site pricing scheme and (iii) both pricing schemes. This yields a 3x3 matrix (see table 1) and we need to determine the profit earned by each firm in the nine potential scenarios.

Lemma 1: When each firm offers one and the same pricing scheme, then the Bertrand equilibrium obtains with equilibrium prices and profits equal to zero.

The proof is straightforward. If the firms select identical pricing schemes in the first stage then in the second stage the pricing game reduces to the Bertrand game. Buyers purchase from the seller that offers the lower price. Any positive price cannot be sustained in equilibrium as the firms have an incentive undercut each other’s price by a small amount. Hence, the Bertrand equilibrium obtains.

Lemma 2: When one firm offers both pricing schemes, then irrespective of the choice of pricing scheme.
scheme of the other firm, the Bertrand equilibrium obtains.

Again the proof is simple: for ease of exposition, lets firm A offer both unit and site licensing schemes and firm B offer only the unit pricing scheme. In the pricing subgame (second stage), the firms compete directly in the unit pricing scheme (offered by both firms). Note that for any positive unit price offered by B, A has can improve its profits by offering a unit price that is marginally below B’s price. This leads to Bertrand price competition in the unit pricing scheme as each firm has an incentive to undercut the other as long as prices are strictly positive (lemma 1). This causes unit prices to fall to zero. When unit prices are zero, all buyers can buy unlimited number of licenses at no cost, hence none of the buyers purchase site licenses from firm A for a strictly positive price. Therefore neither firm earns any profits. The argument is easily extended to the case where B offers only the site pricing scheme while A offers both pricing schemes.

**Table 1: Seller’s profit from adopting various pricing schemes (PS)**

<table>
<thead>
<tr>
<th>Pricing Scheme (PS) Chosen by Firm A</th>
<th>Unit PS</th>
<th>Site PS</th>
<th>Both PS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm B Unit PS</td>
<td>(0,0)</td>
<td>(( \pi_u, \pi_s ))</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Firm B Site PS</td>
<td>(( \pi_u, \pi_s ))</td>
<td>(0,0)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Firm B Both PS</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

Now consider the case where each firm offers a single pricing scheme that is different from its competitor. In particular, we are interested in knowing whether there exist conditions under which the firms can earn strictly positive profits. When the firms can earn profits that are strictly positive and their prices constitute an equilibrium in the pricing subgame then such an equilibrium is clearly Pareto Dominant. Proposition 3 establishes conditions under which such an equilibrium exists and the firms charge positive prices and yet have no incentive to deviate to a lower price. The optimal prices of the sellers may be equal or less than the optimal price for the respective monopolist. At these prices buyers obtain equal or greater surplus relative to their surplus under a monopoly. The following proposition summarizes the conditions, optimal prices and profits for this Pareto-dominant equilibrium.
Proposition 3. The following is the Pareto-dominant pure strategy equilibrium in the duopoly game:

1. \( p_u^* = \omega, \quad p_s^* = \alpha^2/2, \quad \pi_u^* = (1 - k)\omega/2, \quad \pi_s^* = k\alpha^2/2 \) when \( \omega > \alpha \) and \( \omega < \alpha^2/2 \) and (1a)

   either \( k > \min \{1/2, \alpha - \omega \alpha \} \) or \( \omega < 2(1 - k)\alpha^2 \) and (1b) either \( k < \max \{1/2, 1/2 \alpha + 4 \omega \} \) or

   \( \omega > \frac{(1 + k(2\alpha - 1))^2}{8k(1 - k)} \)

2. \( p_u^* = \alpha^2/2, \quad p_s^* = \alpha^2/2, \quad \pi_u^* = (1 - k)\alpha^2/4, \quad \pi_s^* = k\alpha^2/2 \) when \( \omega > \alpha \) and \( \alpha > 2 \) and \( k \leq \frac{1}{1 + 2\alpha(\alpha - \sqrt{\alpha(\alpha - 2)})} \).

Proof. Please see Appendix A.3 for proof.

Proposition 3 lays out the conditions under which the duopolists can set strictly positive prices in equilibrium. The proof of the proposition establishes that even though both firms have the choice of unilaterally deviating from this solution, neither finds it optimal to do so. Each firm finds it optimal to ‘specialize’ by offering a pricing scheme that targets a particular segment of buyers and thus avoid head-to-head competition that leads to the Bertrand equilibrium. Such an equilibrium is sustainable because the duopolists, with their different pricing schemes are no longer perfect substitutes. The pricing schemes induce different levels of usage among different buyer types which lead to differences in their willingness to pay for the two pricing schemes. We elaborate and formalize these arguments below in §2.3.

In order to understand the contribution of this section, it is useful to browse section 5.2 of Tirole [1988] which lists three ways of avoiding the Bertrand equilibrium: the Bertrand outcome can be avoided when sellers are capacity constrained (Edgeworth 1897); when products are differentiated (d’Aspremont, Gabszewicz, and Thisse 1979); when the model is changed from one of short-run competition to long-run competition (Chamberlin 1929). In addition we know that in the short-run, firms can avoid the Bertrand equilibrium when consumers are not perfectly informed or if it is costly for them to obtain information (Salop 1977). In this section, we established that firms can avoid the Bertrand equilibrium by using asymmetric pricing schemes.
2.3 Discussion

How can pricing schemes be used as a tool for differentiation? There are several aspects of the role of pricing schemes that are noteworthy. First note that the amount consumed by a buyer may depend on the price and the marginal benefit from an additional unit of consumption can vary across consumers. For example, most consumers are likely to use more mobile phone minutes if they are cheaper or if they are on a fixed price plan that allows unlimited usage. We know from anecdotal and empirical evidence that this demand elasticity varies across consumers. Moreover aggregate demand elasticity is likely to be affected by the pricing scheme and vary across buyer segments. Jain et al. [1999] present data from an experiment in the context of the cellular phone service showing different buyer segments. They refer to the high usage segment as the Business/Professional segment and the low usage segment as the Personal segment and find that the Professional segment is more sensitive to the calling rate while the personal segment places greater weight on the monthly fee. Danaher [2002] presents the results of a field experiment with residential phone customers finding support for two segments among the residential consumers. They also report demand elasticity and attrition elasticity for the two segment model that are consistent with our model.

Pricing schemes can be designed to fit buyers’ demand patterns so as to maximize the benefit and usage while minimizing the probability that they may be priced out of the market. For example a low usage buyer is likely to prefer a pricing scheme with metered usage while a buyer with high expected demand is likely to prefer a fixed fee pricing scheme. Another example is the ‘family calling plan’ offered by cell phone providers that allow family members to pool their credits and offers unlimited calling within the family. This notion of fitting a pricing scheme to buyer demand is similar to firms’ designing product features for distinct buyer segments and opens the way for pricing schemes to be used as a tool for differentiation.

To provide greater insight into the aggregate demand elasticity and how it varies across C and D type buyers, it is useful to look at aggregate unit demand as a function of price. This is shown in figure 2. Consistent with the findings of Danaher [2002], the reduction in demand from C-type buyers with increases in the site price are due to higher attrition as an increasing number of C-type
Figure 2: C-type buyers decreases with unit price while demand from D-type buyers holds steady. On the right, we see quantity demanded by C-type buyers holds constant while the demand from D-type buyers falls with price. \((\omega = 10, \alpha = 5, k = 0.1)\)

buyers are priced out of the markets. Whereas the reduction in demand from D-type buyers with increase in the unit price is due to reduction in usage levels.

The impact of price increases on aggregate demand is reflected in the profit that a firm can earn from C and D-type buyers using unit or site pricing schemes. This is depicted in figure 3, where a monopolist can earn twice the profit from D-type buyers using site pricing compared to unit pricing. The firm earns twice the profit from C-type buyers when using unit pricing scheme compared to the site pricing scheme.

The effect of pricing schemes on aggregate demand and profitability together indicate that the unit pricing scheme seems more suitable for C-type buyers whereas the site pricing scheme is more suitable for D-type buyers. We refer to this notion of ‘fit’ between certain pricing schemes and buyer segments as the congruousness of a pricing scheme with a segment of buyers. We offer two measures of congruousness:

**Definition 1:** Demand Congruousness(PS, BS) =

\[
\frac{\text{Quantity Demanded under Monopoly Pricing(PS, BS)}}{\text{Max Possible Demand(BS)}}
\]

where PS refers to Pricing Scheme (Site or Unit) and BS to Buyer Segment (C or D-segment).

The monopoly prices for each segment for each pricing scheme are easy to calculate: \(p^C_u = \omega\),
Figure 3: Monopoly prices and profits from C and D-type buyers. Site licensing yields higher profit from D-type buyers whereas unit pricing scheme yields higher profit from C-type buyers.

\[ p_u^D = \alpha/2, \quad p_s^C = \omega/2, \quad p_s^D = \alpha^2/2. \]

The demand at these prices is \( q_u^C = (1 - k) \), \( q_u^D = k\alpha/2 \), \( q_s^C = 3(1 - k)/8 \), \( q_s^D = k\alpha \). Thus the demand congruousness (DC) can be calculated as \( DC_u^C = 1 \), \( DC_u^D = 1/2 \), \( DC_s^C = 3/4 \), \( DC_s^D = 1 \). This shows that the unit pricing scheme has higher demand congruousness with C-type buyers while the site pricing scheme has higher demand congruousness with D-type buyers. Demand congruousness ultimately impacts profit, we define congruousness based on profit as:

**Definition 2:** Congruousness(PS, BS) = \[ \frac{\text{Max Monopoly Profits(PS, BS)}}{\text{Max Possible Gains from Trade(BS)}} \]

Since the seller does not incur any marginal or transactions costs in our model, the maximum possible gains from trade is the sum of WTP across all buyers in the relevant segment. The maximum possible gains from trade from C and D-type buyer segments are \((1 - k)\omega/2\) and \(k\alpha^2/2\) respectively. Based on the monopoly prices reported in the previous paragraph we can compute profits: \( \pi_u^C = (1 - k)\omega \), \( \pi_u^D = k\alpha^2/4 \), \( \pi_s^C = (1 - k)\omega/2 \), \( \pi_s^D = k\alpha^2/2 \). Thus the congruousness (C) is \( C_u^C = 1 \), \( C_u^D = 1/2 \), \( C_s^C = 1/2 \), \( C_s^D = 1 \). Consistent with demand congruousness, we find that the unit pricing scheme is congruous with the C-type buyer segment while the site pricing scheme is congruous with the D-type buyer segment. Why is congruousness important? Differences in congruousness lead to differences in competitiveness with respect to different buyer segments so that when a seller limits itself to a particular pricing scheme, it focusses on the buyer segment with higher congruousness with the respective pricing scheme. This creates differentiation between the
sellers\(^4\).

To verify the robustness of our finding in this section, we have also explored variants of the C and D-type buyers demand functions - for instance by modifying the slope or marginal utility of each buyer type. The Pareto-dominant equilibrium with sellers selecting different pricing schemes and earning strictly positive profits continues to exist for a wide range of perturbations. We have also verified that the addition of another pricing schemes – a two part tariff scheme does not change our results presented in proposition 3. We have assumed thus far that the information good being sold is a commodity. Now we explore the use of different pricing schemes when the information good is differentiated.

3 Horizontally Differentiated Duopoly

In §2 we established the existence of an equilibrium where firms selling commoditized information goods can make substantial profits by using asymmetric pricing schemes. What is the role for different pricing schemes when products are differentiated? Should pricing schemes be used for incremental differentiation? How does the intensity of competition affect the sellers’ choice of pricing schemes? We show in this section that when the products are strongly differentiated it is optimal for competitors to use homogenous pricing schemes, but when products are weakly differentiated, it is optimal to use asymmetric pricing schemes.

We use Hotelling’s [1929] model of horizontal differentiation as modified by d’Aspremont, Gabaszewicz, and Thissse [1979] where buyers are uniformly distributed along the unit line and incur transportation/misfit costs that are a quadratic function of their distance from that of the seller. Horizontal models are appropriate when different buyers prefer products from different vendors even when they are offered at the same price. The misfit cost is a stylized representation of the loss in utility from using a product that is not a perfect fit for the buyer. For example, some buyers

\(^4\)Note that there exist pricing schemes with 100% congruousness for each of the buyer segments – for example if the firms offered both pricing schemes, the congruousness would be 100% for each buyer type. If one firm adopts such a pricing scheme with 100% congruousness across all buyer segments and the competitor’s pricing scheme has 100% congruousness on a subset of the buyers, then the two firms become undifferentiated to that subset of buyers. This leads to the Bertrand outcome, and all other buyers will also take advantage of the resulting prices.
prefer Microsoft’s Windows Operating System (OS) while others prefer Apple’s Mac OS X. A user who prefers Windows OS will have a lower WTP for Mac OS X and vice-versa. Enterprise software vendors often differentiate themselves by specializing their products for specific industry sectors. For example i2 sells supply chain software that has special features for the retail and semiconductor sectors which makes their software a better fit for firms in these industries. The horizontal differentiation model provides a stylized representation of heterogeneity in buyers’ preferences for differentiated products.

Following d’Aspremont, Gabszewicz, and Thisse [1979] and Desai and Purohit [2004], the sellers are located at the ends of the unit line. Representative buyers are uniformly distributed along the line with a maximum willingness to pay $V$. We model the misfit costs incurred by a buyer located at $x$ as $T(x) = t(x - x')^2$ when purchasing from a seller located at $x'$. We have verified that our results hold even when misfit costs are linear instead of quadratic. Following the model presented in §2, we assume that there are two segments of buyers ($S_1$ and $S_2$) and two pricing schemes ($P_1$ and $P_2$). Pricing scheme $P_1$ has a higher congruousness with buyers in segment $S_1$ while pricing scheme $P_2$ has higher congruousness with buyers in segment $S_2$. We use the definition of demand congruousness from §2.3 to model the demand function for each segment of buyers. Consistent with the demand elasticity shown in figure 2, the quantity demanded by buyers from the congruous pricing scheme is constant up to a price $V$, while the demand under the non-congruous pricing scheme decreases as price increases. We represent demand from a buyer belonging to segment $S_1$ under pricing scheme $P_1$ as $D(S_1, P_1)$. For congruous pricing schemes, without loss of generality, we normalize demand to 1. Thus $D(S_1, P_1) = D(S_2, P_2) = 1$ for prices less than or equal to $V$, and zero for prices greater than $V$. Again, consistent with §2.3 and figure 2, demand decreases with price for non-congruous pricing schemes: $D(S_1, P_2) = 1 - \frac{p(P_2)}{V}$ and $D(S_2, P_1) = 1 - \frac{p(P_1)}{V}$ for prices less than or equal to $V$, and zero for prices greater than $V$. Note that we use $p(P_1)$ and $p(P_2)$ to represent the price under pricing scheme $P_1$ and $P_2$ respectively. The surplus for a buyer located at $x$ in segment $S_1$ under pricing scheme $P_1$ can be expressed as $S(S_1, P_1) = (V \cdot D(S_1, P_1) - T(x) - p(P_1)) = (V - T(x) - p(P_1))$ and for pricing scheme $P_2$ as $S(S_1, P_2) = (V \cdot D(S_1, P_2) - T(x) - p(P_2))$. 

\[ S(S_1, P_1) = (V - T(x) - p(P_1)) \]
\[ S(S_1, P_2) = (V - T(x) - p(P_2)) \]
\[ V \left( 1 - \frac{p(P_2)}{V} \right) - T(x) - p(P_2). \]
Similarly the surplus of buyers in segment \( S_2 \) can be stated as
\[ S(S_2, P_1) = V \left( 1 - \frac{p(P_1)}{V} \right) - T(x) - p(P_1) \]
and \( S(S_2, P_2) = (V - T(x) - p(P_2)) \). We assume that buyers’ valuation is sufficiently greater than the transportation costs so that the market is always covered\(^5\). Each firm can choose to offer either one or both of the two pricing schemes.

Maintaining the formulation in §2, the sellers decide which pricing schemes to offer in the first stage of the game and the optimal prices in the second stage of the game. To obtain the equilibrium, we first solve the second stage of the game for each of the possibilities in the first stage. Hence we will solve for optimal prices in the following four scenarios: (i) Each seller offers both pricing schemes (ii) Each seller offers one (same) pricing scheme (iii) Each seller offers one (different) pricing scheme (iv) One seller offers both while the other offers one pricing scheme. In each of these scenarios more than one equilibrium may exist. For example, the social welfare maximizing solution with all prices set to zero is an equilibrium in many of the cases analyzed below. We focus on the Pareto dominant equilibrium that maximizes the sum of the profits earned by the two sellers.

### 3.1 Each Seller offers both Pricing Schemes

We analyze the second stage pricing subgame for the case where in the first stage, both sellers choose to offer both pricing schemes. Therefore seller A offers pricing scheme \( P_1 \) and \( P_2 \). We represent this as \( P^A_1, P^A_2 \). Similarly seller B offers both pricing schemes represented as \( P^B_1, P^B_2 \).

For brevity, we use the notation \( P^A_1 \) for price charged under pricing scheme \( P^A_1 \) instead of \( p(P^A_1) \). Thus \( P^A_1 \) represents both the seller’s choice of a pricing scheme and the price under that pricing scheme. Similarly \( P^A_2, P^B_1, P^B_2 \) replace \( p(P^A_2), p(P^B_1), p(P^B_2) \) respectively. Let \( x_1 \) be the location of the buyer who is indifferent between \( P^A_1 \) and \( P^B_1 \) and similarly \( x_2 \) be the location of the buyer who is indifferent between \( P^A_2 \) and \( P^B_2 \). The indifference equations are

\(^5\)The condition \( V \geq 19t/12 \) is sufficient to guarantee market coverage and henceforth we assume that this condition holds.
Figure 4: Seller A is located at $x = 0$ and offers pricing schemes $P_1^A$ and $P_2^A$. Seller B is located at $x = 1$ and offers pricing schemes $P_1^B$ and $P_2^B$. There are two segments of buyers $S_1$ and $S_2$.

When each seller offers both pricing schemes, the buyer segments are split between the two sellers as shown in figure 4.

Solving for the two indifference point, $x_1$ and $x_2$, we obtain $x_1 = \frac{P_B - P_A + t}{2t}$, $x_2 = \frac{P_B - P_A + t}{2t}$.

The profit functions can be written as

$$
\pi^A = x_1 \cdot P_1^A + x_2 \cdot P_2^A, \quad \pi^B = (1 - x_1)P_1^B + (1 - x_2)P_2^B
$$

Substituting for $x_1$ and $x_2$ and solving first order conditions (FOC) (see appendix A.4: $\frac{\partial \pi^A}{\partial P_1^2} = 0, \frac{\partial \pi^A}{\partial P_1^1} = 0, \frac{\partial \pi^B}{\partial P_1^1} = 0, \frac{\partial \pi^B}{\partial P_2^2} = 0$), we obtain the following optimal solution:

**Proposition 4:** When sellers offer both pricing schemes in the first stage then the following is the Pareto-dominant equilibrium in the second stage: the equilibrium prices are $P_1^A^* = P_2^A^* = P_1^B^* = P_2^B^* = t$. The indifference points are $x_1^* = x_2^* = 1/2$ and the sellers’ optimal profit is $\pi^A^* = \pi^B^* = t$.

Note that $P_1^A^* = P_2^A^* = P_1^B^* = P_2^B^* = 0$ is also an equilibrium but is Pareto-dominated by the equilibrium stated in proposition above. We have verified that the individual rationality
Figure 5: Sellers A and B offer pricing scheme $P_1$. The buyers in each segment split at the indifference points and purchase from different sellers.

and incentive compatibility constraints are satisfied for all buyers. Consistent with d’Aspremont, Gabszewicz, and Thisse [1979] and section 7.1 of Tirole [1988], we find that equilibrium prices and profits are increasing functions of $t$, which captures the level of differentiation between the sellers. Recall that the misfit costs $T(x) = t(x - x')^2$, therefore when $t$ is larger, buyers located close to a seller are less likely to switch to the competing seller. Thus sellers can charge higher prices and benefit from increasing differentiation.

3.2 Each Seller Offers One Pricing Scheme

When each seller offers a single pricing scheme in the first stage, the pricing schemes they offer could be identical ($\{P_A^1, P_B^1\}$ or $\{P_A^2, P_B^2\}$) or they could be different ($\{P_A^1, P_B^2\}$ or $\{P_A^2, P_B^1\}$). We solve the second stage pricing subgame for each of these two cases below.

3.2.1 The Two Sellers Offer The Same Pricing Scheme

When both sellers offer one and the same pricing scheme either $\{P_A^1, P_B^1\}$ or $\{P_A^2, P_B^2\}$, the buyer segments split between the two sellers as shown in figure 5. We analyze the case where both sellers adopt $P_1$. The case where they adopt $P_2$ can be obtained from symmetry.

The indifference equations are
\[ V - P_1^A - tx_1^2 = V - P_1^B - t(1 - x_1)^2 \]
\[ V \left( 1 - \frac{P_1^A}{V} \right) - P_1^A - tx_2^2 = V \left( 1 - \frac{P_1^B}{V} \right) - P_1^B - t(1 - x_2)^2 \]

Solving for \( x_1 \) and \( x_2 \), we obtain \( x_1 = \frac{P_1^B - P_1^A + t}{2t} \), and \( x_2 = \frac{2P_1^B - 2P_1^A + t}{2t} \). The profit functions can be written as

\[ \pi^A = x_1 \cdot P_1^A + x_2 \cdot P_1^A, \quad \pi^B = (1 - x_1)P_1^B + (1 - x_2)P_1^B \]

Substituting for \( x_1 \) and \( x_2 \) and solving FOCs (see appendix A.4: \( \frac{\partial \pi^A}{\partial P_1^A} = 0, \frac{\partial \pi^B}{\partial P_1^B} = 0 \)), we obtain the following optimal solution:

**Proposition 5:** When sellers offer one and the same pricing scheme in the first stage then the following is the Pareto-dominant equilibrium in the second stage: The equilibrium prices are \( P_1^{A*} = P_1^{B*} = 2t/3 \). The indifference points are \( x_1^* = x_2^* = 1/2 \) and the sellers’ profit is \( \pi^{A*} = \pi^{B*} = 2t/3 \).

Note that \( P_1^{A*} = P_1^{B*} = 0 \) is also an equilibrium but is Pareto-dominated by the equilibrium stated in proposition above. The profits are lower in proposition 5 relative to proposition 4 because there is lower demand from segment \( S_2 \), since they are not offered pricing scheme \( P_2 \) which is more congruous with their usage pattern.

### 3.2.2 The Two Sellers Offer Different Pricing Schemes

When the two sellers offer different pricing schemes either \( \{P_1^A, P_2^B\} \) or \( \{P_2^A, P_1^B\} \), there are two potential equilibria. One possibility is that each seller dominates the buyer segment that prefers its pricing scheme. The other possibility is that each seller attracts some buyers from each segment. We analyze each of these cases below.

**Case (i): Each buyer segment purchases from the seller offering the congruous pricing scheme:** We solve the case where the two sellers offer \( P_1^A \) and \( P_2^B \). Each seller can potentially
Figure 6: Seller A offers pricing scheme $P_1$ and B offers $P_2$. There are two potential equilibria: (i) A covers buyer segment $S_1$ while B covers segment $S_2$ and (ii) Each segment is split between the two sellers.

dominate the segment of buyers that has higher congruousness with its pricing scheme. The profit functions can be written as $\pi^A = x_1 \cdot P_1^A$ and $\pi^B = (1 - x_2)P_2^B$. The indifference points are $x_1 = 1$ and $x_2 = 0$. The optimal prices are obtained by determining the highest price such that each firm covers its buyer segment. We find that this is indeed an equilibrium. Neither firm has an incentive to deviate from the high prices that are sustained in this equilibrium and buyers obtain a higher surplus from buying from the seller with the congruous pricing scheme.

**Proposition 6:** When seller A offers pricing scheme $P_1$ and B offers $P_2$, and $V \geq 2t$ then the equilibrium prices are $P_1^A^* = P_2^B^* = V - t$. The indifference points are $x_1^* = 1$, $x_2^* = 0$ and the sellers’ optimal profit is $\pi^A^* = \pi^B^* = V - t$.

**Proof.** Please see Appendix A.5 for proof. ■

The profits earned by the sellers under proposition 6 are quite large. When $V > 2t$, the profits are larger than those in proposition 4 and 5. In §3.4, we will show that the pricing schemes and prices reported in proposition 6 constitute an equilibrium of the two stage game under certain conditions. These profits can be large because, each seller is offering a pricing scheme that is distinct from its competitor. The buyers obtain greater usage from the congruous pricing scheme and are thus willing to pay more for it. Since buyers are less willing to switch to the non-congruous pricing scheme, the two sellers become more differentiated from each other and can sustain higher prices.
Case (ii): Buyer segments are split between the two sellers: When each of the buyer segments are split between the two sellers the indifference equations are

\[ V - P_A^1 - tx_1^2 = V \left(1 - \frac{P_B^2}{V}\right) - P_B^2 - t(1 - x_1)^2 \]

\[ V \left(1 - \frac{P_A^1}{V}\right) - P_A^1 - tx_2^2 = V - P_B^2 - t(1 - x_2)^2 \]

Solving for \( x_1 \) and \( x_2 \), we obtain \( x_1 = \frac{2P_B^2 - P_A^1 + t}{2t} \), and \( x_2 = \frac{P_B^2 - 2P_A^1 + t}{2t} \). The profit functions can be written as

\[ \pi^A = x_1 \cdot P_A^1 + x_2 \cdot P_A^1, \quad \pi^B = (1 - x_1)P_B^2 + (1 - x_2)P_B^2 \]

Solving FOCs (see appendix A.4: \( \frac{\partial \pi^A}{\partial P_1^A} = 0, \frac{\partial \pi^B}{\partial P_2^B} = 0 \)) we obtain the following solution that could potentially constitute an equilibrium:

Prices are \( \tilde{P}_A^1 = \tilde{P}_2^B = 2t/3 \) and the sellers’ profit is \( \tilde{\pi}^A = \tilde{\pi}^B = 2t/3 \). To verify whether this solution is a Nash equilibrium we check whether a firm, say A, has an incentive to unilaterally deviate by making a discrete increase in its price, and thereby abandoning the non-congruous buyer segment \((S_2)\). We redefine \( \pi^A \) by substituting \( P_B^2 = 2t/3 \), thus obtaining \( \pi^A = \left(\frac{\tilde{P}_1^A - P_A^1}{2t}\right) P_A^1 \). We now solve \( \frac{\partial \pi^A}{\partial P_1^A} = 0 \) to obtain \( \tilde{P}_1^A = \frac{7t}{6} \) and yielding a profit of \( \tilde{\pi}^A = \frac{49t}{72} > \tilde{\pi}^A = 2t/3 \). Hence the solution indicated by \( \tilde{P}_1^A = \tilde{P}_2^B = 2t/3 \) is not a Nash equilibrium and sellers will find it profitable to deviate unilaterally.

3.3 Firm A Offers Both Pricing Schemes and Firm B Offers Only One Pricing Scheme

Now we consider the subgame where seller A offers pricing schemes \( P_1 \) and \( P_2 \) while seller B offers only \( P_2 \). The case where B offers both pricing schemes and A offers only one can be obtained by symmetry. Similar to the subgame where each seller offered a single (different) pricing scheme, the buyer segments can (potentially) be either dominated by a single seller or split between the two
sellers. Therefore we solve two separate cases below.

### 3.3.1 Case (i): Each buyer segment is split between the two sellers

When each buyer segment is split between the two sellers, the indifference equations are

\[
V - P_1^A - tx_1^2 = V \left(1 - \frac{P_2^B}{V}\right) - P_2^B - t(1 - x_1)^2
\]

\[
V - P_2^A - tx_2^2 = V - P_2^B - t(1 - x_2)^2
\]

Solving for \(x_1\) and \(x_2\), we obtain \(x_1 = \frac{2P_2^B - P_1^A + t}{2t}\), and \(x_2 = \frac{P_2^B - P_2^A + t}{2t}\). The profit functions can be written as

\[
\pi^A = x_1 \cdot P_1^A + x_2 \cdot P_2^A, \quad \pi^B = (1 - x_1)P_2^B + (1 - x_2)P_2^B
\]

Substituting for \(x_1\) and \(x_2\) and solving FOCs (see appendix A.4: \(\frac{\partial \pi^A}{\partial P_1^A} = 0, \frac{\partial \pi^A}{\partial P_2^A} = 0, \frac{\partial \pi^B}{\partial P_2^B} = 0\)), we obtain the following optimal solution:

**Proposition 7**: When seller A offers pricing scheme \(P_1\) and \(P_2\) and B offers only \(P_2\), then the optimal equilibrium prices are \(P_1^A^* = \frac{7t}{56}, P_2^A^* = \frac{5t}{56}, P_2^B^* = \frac{2t}{3}\). The indifference points are \(x_1^* = \frac{7}{12}, x_2^* = \frac{5}{12}\). Seller A’s profit is \(\pi^A^* = \frac{37t}{360}\) and B’s profit is \(\pi^B^* = \frac{2t}{3}\).

**Proof.** Please see Appendix A.6 for proof. 

We have verified that buyers individual rationality and incentive compatibility constraints are satisfied. We have also verified that B does not have an incentive to deviate unilaterally by reducing prices and increasing its market share.

Proposition 7 is useful in determining the equilibrium in the first (pricing scheme selection) stage of the game. The profit earned by seller A, determine whether and under what conditions the solutions in propositions 4, 5 and 6 are an equilibrium of the complete two stage game. We will do this analysis in §3.4.
Buyer Segment $S_1$

A  

B  

Buyer Segment $S_2$

P_1^A  

P_2^A  

P_1^B  

P_2^B  

$P_1$  

P_2  

$P_1$  

P_2  

Figure 7: Seller A offers pricing scheme $P_1$ and $P_2$ while B offers only $P_2$. Only the solution on the left is a subgame equilibrium with both segments $S_1$ and $S_2$ split between the two sellers.

3.3.2 Case (ii) Buyer segment $S_1$ purchases from A, while segment $S_2$ is split between sellers A and B

Since both sellers offer pricing scheme $P_2$, the buyer segment $S_2$ will be split between the two sellers. However, it is possible that under certain conditions seller A may be able to cover segment $S_1$. We compute the candidate solution and test whether either firm has an incentive to deviate. We find that firm B has an incentive to reduce its price and increase market share, this the solution presented below is not an equilibrium. We provide the setup of the model briefly starting with the indifference equations:

\[ V - P_1^A - tx_1^2 = V \left(1 - \frac{P_2^B}{V}\right) - P_2^B - t(1 - x_1)^2 \]

\[ V - P_2^A - tx_2^2 = V - P_2^B - t(1 - x_2)^2 \]

Solving for $x_1$ and $x_2$, we obtain $x_1 = \frac{2P_2^B - P_1^A + t}{2t}$, and $x_2 = \frac{P_2^B - P_1^A + t}{2t}$. We know from our setup that $x_1 = 1$, solving for $P_1^A$ we get $\hat{P}_1^A = 2P_2^B - t$. The profit functions can be written as

\[ \pi^A = x_1 \cdot \hat{P}_1^A + x_2 \cdot P_2^A, \quad \pi^B = (1 - x_1)P_2^B + (1 - x_2)P_2^B \]

Substituting for $x_1$ and $x_2$ and solving FOCs (see appendix A.4: $\frac{\partial \pi^A}{\partial P_2^A} = 0$, $\frac{\partial \pi^B}{\partial P_2^B} = 0$), we obtain the following solution: prices are $\hat{P}_1^A = \hat{P}_2^A = \hat{P}_2^B = t$, seller A’s profit is $\hat{\pi}_1^A = 3t/2$ and B’s profit.
Note that $B$ has an incentive to reduce price and target both segments of buyers since the profit by doing so as stated in proposition 7 ($2t/3$) is greater than $\tilde{\pi}^B = t/2$. Hence the solution stated above is not an equilibrium.

### 3.4 Equilibrium Selection of Pricing Schemes

Recall that the firms play a two stage game where they determine their pricing scheme in stage 1 and the optimal prices in stage 2. In §3.1 – §3.3, we solved the second stage of the game determining equilibrium prices in each of the subgames. Now we determine the equilibrium in the first stage of the game to identify the pricing schemes that will be adopted in equilibrium. We introduce some notation to represent each potential pricing scheme that may be offered in stage 1. We represent the set of pricing schemes that are offered by each firm in the first stage within brackets, for example, \{ $P^A_1, P^A_2, P^B_1, P^B_2$ \} represents the case where firms A and B offer both pricing schemes ($P_1$ and $P_2$). We can now use the equilibrium profits from propositions 4, 5, 6 and 7 to identify the Pareto dominant equilibrium for the overall game (across the two stages). The following proposition states the equilibrium for the two stages of the game:

**Proposition 8:** Pareto-dominant equilibria for the two-stage game:

- **When** $t \leq 36V/73$ **then** \{ $P^A_1, P^B_2$ \} and \{ $P^A_2, P^B_1$ \} **are Pareto dominant equilibria.** The optimal prices are $P^A_{1*} = P^B_{2*} = V - t$ and the optimal profit is $\pi^A_{1*} = \pi^B_{2*} = V - t$.

- **When** $36V/73 < t \leq 12V/19$ **then** \{ $P^A_1, P^A_2, P^B_1, P^B_2$ \} **is the Pareto dominant equilibrium.** The optimal prices are $P^A_{1*} = P^A_{2*} = P^B_{1*} = P^B_{2*} = t$. The optimal profit is $\pi^A_{1*} = \pi^B_{2*} = t$.

**Proof.** Recall that $V \geq 19t/12$ is sufficient to ensure market coverage of both segments. Comparing profits across propositions 4 - 7, we find that when $t$ is sufficiently small relative to $V$, proposition 6 yields the highest profit with \{ $P^A_1, P^B_2$ \} or \{ $P^A_2, P^B_1$ \}. Consider the incentive for either firm (say A) to unilaterally deviate from \{ $P^A_1, P^B_2$ \}. Seller A could add pricing scheme $P^A_2$ thus deviating to \{ $P^A_1, P^A_2, P^B_2$ \} resulting in a profit of $\pi_A = 37t/36$ as stated in proposition...
7. Comparing to a profit of $V - t$ in proposition 6, we find that such a deviation is profitable for A when $t > 36V/73$. When $t$ is sufficiently small $t \leq 36V/73$, then the equilibrium stated in proposition 6 with each firm offering one pricing scheme that is different from its competitor is an equilibrium of the 2 stage game.

Now consider the case where $t > 36V/73$. We showed that seller A has an incentive to deviate from $\{P^A_1, P^B_2\}$ to $\{P^A_1, P^A_2, P^B_2\}$. Now seller B would benefit by adding pricing scheme $P^B_1$ leading to $\{P^A_1, P^A_2, P^B_1, P^B_2\}$ (from proposition 4 and proposition 7). Thus if we consider the equilibrium stated in proposition 4 with $\{P^A_1, P^A_2, P^B_1, P^B_2\}$, we can see that when $t > 36V/73$, neither firm has an incentive to unilaterally deviate by reducing the set of pricing schemes being offered. Hence proved.

Proposition 8 pulls together all the propositions in this section and establishes the Pareto-dominant equilibrium of the two stage game. It shows that when the seller are highly differentiated ($t > 36V/73$) they should offer both pricing schemes. This strategy is optimal for well differentiated sellers because the loss of incremental differentiation from the use of distinct pricing schemes is relatively small for sellers when they are already well differentiated. They are able to target both segments of buyers by offering both pricing schemes so that buyers can self-select the pricing scheme that is congruous to their usage pattern. In contrast when the sellers are weakly differentiated, the incremental differentiation provided by the use of distinct pricing schemes helps to mitigate price competition and improves profits.

Proposition 8 also extends the region over which an asymmetric equilibrium in pricing schemes has been shown to exist. In contrast to Jain and Kannan [2002], we have shown that asymmetric pricing schemes can be sustained in equilibrium even when marginal costs are zero. Comparing to Sundararajan’s [2004b] analysis of a monopoly, proposition 8 shows that it can be optimal for a duopoly to offer a fixed fee contract in addition to a usage based contract even in the absence of transactions costs. In the discussion following proposition 2, we also showed that a monopolist will also offer both pricing schemes under certain conditions.
4 Discussion

We seek to explain the observed diversity in pricing schemes for information goods and whether sellers can use pricing schemes as a strategic tool to position themselves in the marketplace. While much of the prior research in pricing of information goods has sought to determine the optimal pricing scheme, this paper shows that in a competitive environment, pricing schemes can be viewed as a strategic tool for differentiation and for targeting customer segments. To the best of our knowledge this is the first analytical paper that examines the role of competition on the selection of pricing schemes for information goods with negligible marginal costs. Recent work has shown that it is optimal for a monopolist to offer multiple pricing schemes — usage and flat fee pricing schemes (Sundararajan [2004b]) in the presence of positive transactions costs. This leaves open the question as to why competing sellers would choose to offer different pricing schemes for the same information good. For example, Apple’s iTunes music download service offers only one pricing scheme — a fixed price for each song. In contrast, Yahoo’s competing music download service also offers only one pricing scheme – subscription service with a monthly fee for unlimited downloads. In the market for Customer Relationship Management (CRM) Software, Salesforce.com offers only one pricing scheme – subscription based pricing whereas Microsoft offers a competing CRM solution (Microsoft Dynamics CRM) but it is not available under the subscription pricing scheme.

We showed in §2 that certain pricing schemes are more congruous to certain buyer segments relative to other pricing schemes. When a seller offers a pricing scheme to a buyer segment that is congruous to their usage pattern, then the buyers’ aggregate unit demand is higher and relatively inelastic. This causes the buyer’s aggregate WTP to be higher for congruous pricing schemes leading to higher revenues and profits for sellers. Using this property, we show in proposition 3 that undifferentiated sellers can sustain equilibrium prices that are strictly positive thus avoiding the Bertrand equilibrium. This is a novel use of pricing schemes and extends the list of previously known ways to avoid the Bertrand equilibrium (see §5.2 of Tirole [1988]). We have shown that sellers may adopt asymmetric pricing schemes thereby earning substantial profits from commoditized information goods, even with perfectly rational, fully informed buyers, no marginal costs and no
transactions costs. Our results have some regulatory implications as we have shown in part (1) of proposition 3 that under certain conditions duopolists can monopolize certain buyer segments.

In §3, we extend the model to the case of information goods that are horizontally differentiated and show that the level of product differentiation plays a role in the selection of pricing schemes. Sellers will use different pricing schemes when their products are not well differentiated whereas in a monopolistic market or a market with well differentiated duopolies, the sellers will use the same set of pricing schemes. We demonstrated the existence of an equilibrium with asymmetric pricing schemes when marginal costs are zero. This result is counter to Jain and Kannan [2002] who showed that such an equilibrium can exist in a model of risk averse consumers who are uncertain about their demand but only when marginal costs are strictly positive. The results in §3 also help explain the observed diversity in pricing schemes offered by information goods vendors.

This paper shows that there are similarities between pricing schemes and product attributes that are used for horizontal differentiation. Pricing schemes can sometimes be used to increase differentiation just like any other product attribute. However, one important difference between pricing schemes and other product attributes is that the principle of maximal differentiation which applies to product attributes does not always apply to pricing schemes. In particular, we showed in proposition 8, that when the sellers are sufficiently differentiated, it is optimal for each seller to offer both pricing schemes. Thus it is sometimes optimal for sellers to be minimally differentiated on the pricing scheme dimension by offering an identical set of pricing schemes.

Our results help managers understand the implications of modifying the pricing schemes that are offered and the link between pricing schemes and the target market. For example Sprint’s *Incoming Free* pricing scheme charges a higher per minute rate for outgoing calls but incoming calls are free. This makes it more attractive to users whose calling pattern involves mostly incoming calls. T-Mobile is currently running an advertising campaign for a similarly innovative pricing scheme called *myFaves* which allows users to make free unlimited calls to five people irrespective of their carrier. This calling plan is unique to *T-Mobile* and makes it more attractive to a segment of users who make a lot of calls to five or less people. Our results also show that the addition of a pricing scheme
that does not focus on a niche segment of buyers is likely to intensify price competition. Thus pricing schemes need to be crafted carefully with a focus on the targeted buyers. Sellers that have highly differentiated products and market power are more likely to be able to successfully offer a range of pricing schemes. New or smaller firms that lack a distinct brand and those who wish to target a niche market are likely to offer a limited set of pricing schemes. This can be tested empirically in the future. There is anecdotal evidence that supports our findings, for example, Oracle is the market leader in the CRM market and offers a number of different pricing schemes. Similarly Microsoft offers a range of pricing schemes for their dominant Windows operating system. On the other hand Salesforce.com, a smaller CRM vendor offers only one pricing scheme; iGate Corp. offers a single pricing scheme for business process outsourcing; Websense.com, a software firm in the Employee Internet Management market offers only per-seat pricing while its competitor Apreo.com offers a fixed price for 100 licenses.

We conclude in the following section by listing some limitations of our study and directions for future research.

5 Conclusion

We have analyzed a duopoly market for a commoditized information good and then extended results to horizontally differentiated sellers. Our results offer a potential explanation for the observed heterogeneity in pricing schemes. We have found that our results are robust to perturbations in the valuation function of the buyers as well as to the addition of certain pricing schemes in the strategy space of the sellers. While we model the degree of differentiation as exogenously determined, in the real world, firms may find it optimal to further tweak product features to fit the buyer segment that is selected based on their pricing scheme. Given the stylized nature of the model, we examined a limited set of factors whereas many other factors are likely to affect the set of pricing schemes offered by a firm in the real world. Future extensions can examine the impact of other factors such as product line and the entry and exit of competing sellers. Our results can also be used to generate empirically testable hypotheses that can be the subject of future research.
References


A Appendix

A.1 Proof of Proposition 1

Proof. The profit function is stated in eq.1 and the interior solution \( \hat{p}_u \) is stated in eq.2. The interior solution \( \hat{p}_u \) is a feasible candidate maxima when \( \hat{p}_u \leq \alpha \) and \( \hat{p}_u \leq \omega \). Solving \( \hat{p}_u \leq \alpha \), we find that \( \hat{p}_u \leq \alpha \), holds when \( k \geq k_{11} \), where \( k_{11} = \frac{1}{1+2\alpha} \). Similarly \( \hat{p}_u \leq \omega \) holds when \( k \geq k_{12} \), where \( k_{12} = \frac{1}{1-2\alpha+4\omega} \). Now we explore the region where \( \hat{p}_u \) is feasible and compare it to the boundary solutions to identify the global maxima. Consider two cases, one where \( \omega \geq \alpha \) and another where \( \alpha > \omega \). When \( \omega \geq \alpha \), \( \hat{p}_u \leq \alpha \iff k \geq k_{11} \) is sufficient to ensure feasible \( \hat{p}_u \). Similarly when \( \alpha > \omega \), \( \hat{p}_u \leq \omega \iff k \geq k_{12} \) is sufficient to ensure feasible \( \hat{p}_u \). To depict various boundary solutions, we use the following notation \( \pi_u(segment(s);pu = boundary) \) for example when \( \omega \geq \alpha \), \( \pi_u(C;pu = \omega) \) indicates the boundary where \( p_u = \omega \), none of the D type buyers can purchase at this price. \( \pi_u(C;pu = \omega) = p_u(\frac{1-k}{2}) \) and \( \pi_u(D;pu) = p_u \cdot k(\alpha - p_u) \) and \( \pi_u(C + D;pu) = \frac{p_u(1+2k\alpha-k-2kp_u)}{2} \) can be obtained from eq.1. Note that \( \hat{p}_u = \frac{\alpha}{2} + \frac{1-k}{4k} \) is greater than \( p_u = \frac{\alpha}{2} \) which maximizes sellers profit from D type buyers. In the region where \( \hat{p}_u \) is feasible, \( \omega \geq \frac{\alpha}{2} + \frac{1-k}{4k} > \frac{\alpha}{2} \). This fact is useful in pruning the number of boundary conditions that need to be compared to \( \pi_u(C + D;pu = \hat{p}_u) = \frac{1+k(2\alpha-1)^2}{16k} \). When \( \omega \geq \alpha \), we compare \( \pi_u(C;pu = \omega) \) to \( \pi_u(C + D;pu = \hat{p}_u) \) and find that \( \pi_u(C;pu = \omega) > \pi_u(C + D;pu = \hat{p}_u) \) when \( \omega > \omega_{11} = \frac{(1+k(2\alpha-1))^2}{8k(1-k)} \) and \( \pi_u(C;pu = \alpha) \leq \pi_u(C + D;pu = \hat{p}_u) \) otherwise. This yields parts (1b) and (2a) of proposition 1. When \( \omega < \alpha \), and \( \hat{p}_u \) is feasible, it is easy to see that \( \pi_u(C + D;pu = \hat{p}_u) \) dominates the boundary \( \pi_u(D;pu = \omega) \). This follows from our prior observation that \( \omega \geq \frac{\alpha}{2} + \frac{1-k}{4k} > \frac{\alpha}{2} \). This yields part (1a) of proposition 1.

Now we consider the region where \( \hat{p}_u \) is not feasible (\( k < Max\{k_{11}, k_{12}\} \)). There are three potential boundaries: \( \pi_u(D;pu = \alpha/2), \pi_u(C;pu = \omega) \) when \( \omega \geq \alpha \) and \( \pi_u(C + D;pu = \omega) \) when \( \omega < \alpha \). First consider the case where \( \omega \geq \alpha \), comparing \( \pi_u(D;pu = \alpha/2) \) and \( \pi_u(C;pu = \omega) \) we find that \( \pi_u(C;pu = \omega) > \pi_u(D;pu = \alpha/2) \) when \( k < k_{13} = \frac{2\omega}{\alpha^2 + 2\omega} \). Note that \( k_{13} > k_{11} \) and \( k_{13} > k_{12} \), thus \( k < k_{13} \) is true in this region (since \( k < Max\{k_{11}, k_{12}\} \)). This yields part (2b) of proposition 1. When \( \omega < \alpha \), comparing \( \pi_u(D;pu = \alpha/2), \pi_u(C + D;pu = \omega) \) we find that \( \pi_u(C + D;pu = \omega) \geq \pi_u(D;pu = \alpha/2) \) when \( k \geq k_{14} = \frac{2\omega}{2\omega + (\alpha - 2\omega)^2} \) and \( \pi_u(C + D;pu = \omega) < \pi_u(D;pu = \alpha/2) \) otherwise. This yields parts (2c) and (3) of proposition 1. Hence proved. ■
A.2 Proof of Proposition 2

**Proof.** The profit function is stated in eq.3 and the interior solution \( \hat{\pi}_s \) is stated in eq.4. The interior solution \( \hat{\pi}_s \) is a feasible candidate maxima when \( \hat{\pi}_s \leq \alpha^2/2 \) and \( \hat{\pi}_s \leq \omega \). Solving \( \hat{\pi}_s \leq \omega \), we find that \( \hat{\pi}_s \leq \omega \), holds when \( k \leq k_{21} \), where \( k_{21} = \frac{1}{2} \). Similarly \( \hat{\pi}_s \leq \alpha^2/2 \), when \( k \leq k_{22} \), where \( k_{22} = 1 - \frac{\omega}{\alpha^2} \).

Now we explore the region where \( \hat{\pi}_s \) is feasible and compare it to the boundary solutions to identify the global maxima. Consider two cases, one where \( \omega \leq \alpha^2/2 \) and another where \( \omega > \alpha^2/2 \). When \( \omega \leq \alpha^2/2 \), \( k \leq k_{21} \) is sufficient to ensure feasible \( \hat{\pi}_s \). Similarly when \( \omega > \alpha^2/2 \), \( k \leq k_{22} \) is sufficient to ensure feasible \( \hat{\pi}_s \).

When \( \omega > \alpha^2/2 \), and \( \hat{\pi}_s \) is feasible, we compare \( \pi_s(C+D;p_s = \alpha^2/2) \) to \( \pi_s(C+D;p_s = \hat{\pi}_s) \) and find that \( \pi_s(C+D;p_s = \hat{\pi}_s) \geq \pi_s(C+D;p_s = \alpha^2/2) \) when \( \omega > \omega_{21} = 2k\alpha^2(1-k) \) and \( \pi_s(C+D;p_s = \hat{\pi}_s) < \pi_s(C+D;p_s = \alpha^2/2) \) otherwise. This yields parts (1b) and (2a) of proposition 2.

A.3 Proof of Proposition 3

**Proof.** For part (1) of proposition 3: In this case the duopolists charge monopoly prices as stated in propositions 1 and 2. This can arise as an equilibrium when the optimal price charged by the monopoly under unit pricing is such that only C-type buyers purchase under the unit pricing...
scheme: \( p_u^{m*} > \alpha \) (and \( p_u^{m*} \leq \omega \)). Similarly, when \( p_s^{m*} > \omega \) (and \( p_s^{m*} \leq \alpha^2/2 \)) only D-type buyers purchase under the site licensing scheme. We can see that these conditions are satisfied in proposition 1 parts (2a) and (2b) and proposition 2 parts (2a) and (2b). Combining the conditions stated in these propositions, we obtain part (1) of proposition 3. To see that this is an equilibrium in the first stage of the game (selection of pricing scheme), note from table 1, that any deviation that changes the pricing scheme or add another pricing scheme results in zero profits in the second stage. Hence this is an equilibrium. Note that in table 1, the only other equilibrium is for both firms to offer both pricing schemes. This results in no profit to either firm. Therefore the equilibrium in proposition 3, part (1) is Pareto-dominant.

For part (2) of proposition 3: For the purpose of exposition let the duopolists be named S and U, and assume wlog that firm S offers site based pricing scheme while firm U offers unit based pricing scheme. When \( p_s^{m*} = \alpha^2/2 \) and \( \alpha^2/2 < \omega \) then both C and D type buyers can purchase from S. When \( \omega > \alpha \), U must charge a price \( p_u < \omega \) in order to earn a profit. Thus C type buyers have a choice between purchasing from S or U. We will show that under certain conditions U will reduce its price to \( \alpha^2/2 \) and capture the entire C-segment while it is optimal for S to charge \( p_s^* = \alpha^2/2 \) and sell to D-type buyers. Let \( n_i \) be the C-type buyer indifferent between \( p_u \) and \( p_s \). The indifference equation is \( (\omega - p_u)n_i = (\omega - p_s) - n_s \). Solving for \( n_i \) we obtain \( n_i = \frac{p_s - p_u}{p_u} \). U’s revenue from C-type buyers is \( \pi_u(C; p_u) = (1 - k) \int p_u \cdot x \cdot dx \). Substituting \( n_i = \frac{p_s - p_u}{p_u} \) and taking derivative wrt to \( p_u \), we get \( \frac{\partial \pi_u(C; p_u)}{\partial p_u} = -\frac{(1-k)p_u^2}{2p_u} < 0 \). Hence U’s profit from C-segment increases by reducing \( p_u \). This is true in the interior until the boundary \( n_i = 1 \), at which point, U covers the C-type buyers. Setting \( n_i = \frac{p_s - p_u}{p_u} = 1 \), we obtain \( p_u = p_s = \alpha^2/2 \). When \( p_u = \alpha^2/2 > \alpha \), then none of the D type buyers can purchase from U. Further, we need to establish that it is not optimal for U to make a discrete reduction in price to attract D-type buyers. We compare \( \pi_u(C + D; p_u) = \hat{\pi}_u \) to \( \pi_u(C; p_u = \alpha^2/2) \). Solving \( \pi_u(C + D; p_u = \hat{\pi}_u) = \pi_u(C; p_u = \alpha^2/2) \) for \( k \), we find 2 roots: \( k_{31} = \frac{1}{1 + 2\alpha(\alpha - 1 + \sqrt{\alpha(\alpha - 2)})} \), \( k_{32} = \frac{1}{1 + 2\alpha(\alpha - 1 - \sqrt{\alpha(\alpha - 2)})} \). We find that \( \pi_u(C + D; p_u = \hat{\pi}_u) \leq \pi_u(C; p_u = \alpha^2/2) \) when \( k_{31} \leq k \leq k_{32} \). Also \( \hat{\pi}_u < \alpha \) when \( k \geq \frac{1}{1 + 2\alpha} \). Note that \( \frac{1}{1 + 2\alpha} > k_{31} \) when \( \alpha > 2 \) (since \( \alpha^2/2 > \alpha \)). When \( k < \frac{1}{1 + 2\alpha} \), \( \hat{\pi}_u \) is not feasible. Thus when \( k \leq k_{32} \), and \( \alpha \leq \alpha^2/2 \leq \omega \) then \( \pi_u(C; p_u = \alpha^2/2) \) is the global maxima.

Finally we need to find conditions under which S finds it optimal to retain \( p_s^* = \alpha^2/2 \). Note that \( p_s = \alpha^2/2 \) is a local maxima since \( \pi_s(D; p_s = \alpha^2/2 + \epsilon) = 0 \) and \( \pi_s(D; p_s = \alpha^2/2 - \epsilon) < \pi_s(D; p_s = \alpha^2/2) \).
for arbitrarily small $\epsilon$. We need to identify conditions under which $S$ finds it optimal to not reduce $p_s$ to sell to C-type buyers. To compute the profit from a lower $p_s$ that would attract D type buyers and some C-type buyers, we compute $\pi_s(C + D; p_s; p_u = \alpha^2/2)$. The first step is to compute the revenue from C-type buyers. $\pi_s(C; p_s; p_u = \alpha^2/2) = (1-k)p_s(1-n_i)$ where $n_i = \frac{p_s}{p_u}$. Adding revenue from D-type buyers ($k \cdot p_s$), we get $\pi_s(C + D; p_s; p_u = \alpha^2/2)$. Solving FOC w.r.t $p_s$, we find that $\hat{p}_s = \frac{\alpha^2}{4 - 4k}$. This interior candidate maxima is feasible when $k < 1/2$. Now we compare $\pi_s(C + D; p_s = \hat{p}_s; p_u = \alpha^2/2)$ to $\pi_s(D; p_s = p_u = \alpha^2/2)$. We find that $\pi_s(D; p_s = p_u = \alpha^2/2) \geq \pi_s(C + D; p_s = \hat{p}_s; p_u = \alpha^2/2)$ when $k \leq 1/2$ which constitutes the entire region where $\hat{p}_s$ is feasible. Hence $\pi_s(D; p_s = p_u = \alpha^2/2)$ is the global maxima for $S$.

Finally to obtain the conditions listed in proposition 3, part (2), note that $\omega > \alpha$ and $\alpha > 2$ implies $\omega > \alpha^2/2$ hence we get the following conditions: $\omega > \alpha$ and $\alpha > 2$ and $k \leq k_{32}$. ■

### A.4 First Order Conditions for Propositions

#### First Order Conditions for Proposition 4:

$$\frac{\partial \pi^A}{\partial P^A_1} = \frac{P^B - 2P^A_2 + t}{2t}, \quad \frac{\partial \pi^A}{\partial P^A_2} = \frac{P^B - 2P^A_2 + t}{2t}, \quad \frac{\partial \pi^B}{\partial P^B_1} = \frac{P^A - 2P^B_2 + t}{2t}, \quad \frac{\partial \pi^B}{\partial P^B_2} = \frac{P^A - 2P^B_2 + t}{2t}$$

#### First Order Conditions for Proposition 5:

$$\frac{\partial \pi^A}{\partial P^A_1} = \frac{3P^B - 6P^A_2 + 2t}{2t}, \quad \frac{\partial \pi^B}{\partial P^A_1} = \frac{3(P^A_1 - 2P^B) + 2t}{2t}$$

#### First Order Conditions for Proposition 7:

$$\frac{\partial \pi^A}{\partial P^A_1} = \frac{2P^B - 2P^A_2 + t}{2t}, \quad \frac{\partial \pi^A}{\partial P^A_2} = \frac{P^B - 2P^A_2 + t}{2t}, \quad \frac{\partial \pi^B}{\partial P^B_2} = \frac{P^A_1 + P^A_2 - 6P^B + 2t}{2t}$$

### A.5 Proof of Proposition 6

**Proof.** When seller A offers pricing schemes $P^A_1$ and B offers $P^B_2$, we want to determine prices such that A covers segment $S_1$ and B covers segment $S_2$. The lowest WTP of a buyer of $S_1$, is the buyer located furthest from A, at $x_1 = 1$. Her WTP is $V - t$. Similarly for seller B, the lowest WTP for any buyer on segment $S_2$ under pricing scheme $P^B_2$ is $V - t$ (for the buyer located at $x_2 = 0$).

To verify whether these prices satisfy incentive compatibility, we compute the surplus of buyer located at $x_1 = 1$ from $P^B_2 = V - t$: $V \left(1 - \frac{P^B_2}{t}\right) - P^B_2 - t(1-x_1)^2 = 2t - V \leq 0$ when $V \geq 2t$. Hence buyer located at $x_1 = 1$ (and all buyers on $S_1$) will purchase from A. By symmetry, all buyers on $S_2$ will purchase from seller B. We have verified that the individual rationality and incentive compatibility constraints for $x_1 = 1$ can only be satisfied for $V \geq 2t$. 

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Finally, we need verify whether either firm has an incentive to deviate. In this case, the only discrete deviation that can possibly raise profits is to cut prices by a large enough amount to attract buyers from the non-congruous segment. Let us compute the profit to seller A if it deviates unilaterally. Redefine the profit of seller A to include profit from both segments: \( \pi^A = x_1 \cdot P^A_1 + (1-x_2)P^A_2 \) where \( x_1 = 1 \) and \( x_2 = \frac{P^B_2 - 2P^A_1 + t}{2t} \) and \( P^B_2 = V - t \). We determine the optimal price for A by solving FOC \( \frac{\partial \pi^A}{\partial P^A_1} = \frac{V + 2t - 4P^A_1}{2t} = 0 \). This yields \( P^A_1 = (V + 2t)/4 \) and profit \( \pi^A = \frac{(V + 2t)^2}{16t} \). This solution is valid when \( x_2 \leq 1 \iff V \leq 6t \). We compare this profit to the profit in proposition 6 \((V - t)\) and find that when \( 2t \leq V \leq 6t \), \( V - t > \frac{(V + 2t)^2}{16t} \). Hence A will not deviate when \( 2t \leq V \leq 6t \). When \( V > 6t \), A covers the segment \( S_2 \) as well as segment \( S_1 \). Solving \( x_2 = \frac{P^B_2 - 2P^A_1 + t}{2t} = 1 \) after substituting \( P^B_2 = V - t \), we find that \( \pi^A = (V - 2t)/2 \) and \( \pi^A = V - 2t \) which is less than \( V - t \), hence A will not deviate. Hence proved. ■

A.6 Proof of Proposition 7

**Proof.** To verify the incentive compatibility constraint for the buyers we need to make sure that since \( P^A_1 > P^A_2 \), that the \( S_1 \) buyer at \( x_1 = 0 \) does not switch from \( P^A_1 \) to \( P^A_2 \). Surplus for buyer \( x_1 = 0 \) from \( P^A_1 = 7t/6 \) is \((V - 7t/6)\). Surplus for buyer \( x_1 = 0 \) from \( P^A_2 = 5t/6 \) is \((V - 5t/3)\). Note that \((V - 7t/6) > (V - 5t/3)\), hence the incentive compatibility constraint is satisfied.

The prices are obtained by solving first order conditions as stated in appendix A.4. To verify that this is an equilibrium, we need to consider any price changes that can incentivize either firm to deviate. Since the solution was obtained by solving FOC, we know that small price changes are not profit improving for either firm. Thus we need only consider large discrete price changes that change the market share substantially. The only large deviation that has the potential for profit improvement is for firm B to increase prices substantially and abandon the \( S_1 \) segment of buyers.

Seller B’s profit from deviating is \((1 - x_2)P^B_2 \) where \( x_2 = \frac{P^B_2 - P^A_1 + t}{2t} \) and \( P^A_2 = 5t/6 \). The first order condition is \( \frac{11}{12} - \frac{P^B_2}{t} = 0 \). Solving for \( P^B_2 \) we get \( P^B_2 = \frac{11t}{12} \) and \( \pi_B = \frac{121t}{288} \). Note that \( \frac{2t}{3} > \frac{121}{288} \), thus B will not deviate. Thus the solution stated in proposition 7 is an equilibrium. Hence proved. ■